

# Kinetic Theory

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## ABSTRACT

This short paper deals with three aspects of kinetic theory: the hypotheses on which it is based, the calculation of the pressure and absolute temperature of an ideal gas, and the main consequences of the theory. The treatment is accessible to anyone familiar with basic algebra, since calculus is used only marginally; however, a thorough understanding requires some background in physics — for instance, the meaning of absolute temperature, energy, and average value of a quantity. The physical quantities and constants needed along the way are introduced as they arise.

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## Introduction

The development of thermodynamics in the early decades of the 19th century grew out of the integration of experimental observations with theoretical premises, leading to the modern theory of heat, expressed by the first and second laws of thermodynamics. Although the first law fully expresses the equivalence of work and heat by stating that heat is a form of energy, it is essentially empirical in nature: it offers no further explanation of why work and heat are equivalent. This gap, due to the lack of a complete theory of molecular behavior, is filled precisely by kinetic theory.

The conceptual core of kinetic theory is to provide a mechanical and statistical interpretation of thermodynamics, by expressing pressure  $P$  and absolute temperature  $T$  as average values of mechanical quantities (mass, velocity, energy, and so on). By calculating the pressure of a perfect (or ideal) gas<sup>1</sup> and comparing the result with the state equation, it becomes possible to express absolute temperature as a function of molecular motion.

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1 A gas is called perfect or ideal if its behaviour agrees, to a good approximation, with Boyle's and Gay-Lussac's laws; this holds if its temperature is well above its condensation point and the gas is highly rarefied. The state equation of a perfect gas,  $PV = nRT$ , relates pressure, volume, absolute temperature and number of moles. The ideal gas constant is  $R \approx 8.31 \text{ J}/(\text{mol}\cdot\text{K})$ .

## Hypotheses of kinetic theory

Kinetic theory rests on a set of hypotheses about the microscopic structure of matter, grounded in empirical observation. Some of these hypotheses apply to matter in general (including the liquid and solid phases), others only to perfect gases.

1. Matter is made up of minimal units (“molecules”), which can be treated as massive but dimensionless material points. In reality molecules have a complex internal structure, but in the basic version of the theory they are taken to be structureless and volumeless. This means that molecules are assumed to undergo only translational motion, so that rotations and vibrations are ignored. Although not physically accurate, this hypothesis produces an easily tractable model; the corrections needed to account for the molecules’ degrees of freedom and total volume are introduced later.

2. Molecules are animated by a continuous random motion, known as thermal agitation. This applies even to states of matter — solids in particular — that appear, at first glance, to consist of motionless particles; according to the theory, in the solid phase molecules merely oscillate around fixed equilibrium positions. Molecular motion is evident in gases and solutions, since these substances tend to diffuse, that is, to fill the entire volume available to them. The pressure of a gas confined in a container is likewise due to its tendency to occupy all the available space, from which one can infer that pressure originates from molecular impacts on the container walls and depends on the mass and average speed of the molecules.

This hypothesis is confirmed by Brownian motion, named after the botanist R. Brown, who observed it in 1827. Brown found that pollen grains (which are extremely light) suspended in a drop of water appear to move in a continuous, random fashion that can be described as a succession of very short segments. Other kinds of particles, provided they have very small mass and size, show the same behavior when floating in a fluid. The explanation is that the fluid’s own molecules are in continuous, random motion, and transfer some of that motion to the suspended particles by colliding with them.

3. The laws of classical mechanics apply to molecular motion. This is a necessary hypothesis for proceeding with calculations, and it can be justified only *a posteriori*, that is, by comparing the theory’s predictions with the available experimental data.

The following hypotheses apply only to ideal gases. Recall that a gas behaves in a “perfect” or “ideal” way — that is, it satisfies the state equation to a good approximation — when it is highly rarefied, so that its pressure is very low and its temperature is well above its condensation point.

4. Molecules neither collide with nor otherwise interact with one another (this is a simplification; a more accurate theory has to account for the weak intermolecular forces known as Van der Waals forces), so each molecule travels in a straight line at constant speed until it strikes the walls of the container. This is equivalent to assuming that the size of the molecules is negligible compared with the average distance between them, so that the more rarefied a gas is, the closer this hypothesis is to reality.

The empirical basis for this hypothesis is the fact that a gas, especially a very rarefied one, can be strongly compressed. By increasing the pressure one can reduce a gas to a very small fraction of its initial volume (for instance, 1/10,000) before condensation sets in; this shows that the total volume occupied by all the molecules of a gas (the covolume) is very small compared with the volume of the container. In practice, the volume of a single molecule is negligible, since it is obtained as the ratio between the covolume and the number of molecules in a given amount of gas — and one mole contains about  $6.022 \times 10^{23}$  molecules. The volume apparently filled by an ideal gas is therefore almost entirely empty, and can be greatly reduced by increasing the pressure.

5. Collisions between the molecules and the container walls are elastic.

This is perhaps the most problematic hypothesis, yet it has to be assumed, since otherwise the pressure of a gas at constant volume and temperature could not stay constant over time, as everyday experience shows. At the macroscopic scale, collisions are never perfectly elastic, though they can be perfectly inelastic; in practice every macroscopic collision involves at least a small dissipation of energy as heat. So this is the hypothesis that seems hardest to accept when judged against macroscopic phenomena. But the need for the theory to remain consistent with observation forces us to accept that collisions are elastic at the microscopic scale — otherwise we would be led to conclusions that contradict experience itself.

Recall that, by definition, a collision is elastic if the kinetic energy  $\frac{1}{2} m v^2$  is conserved. Suppose, for the sake of contradiction, that the speed of the molecules decreased at every collision with the container walls. Then both the intensity and the frequency (number of collisions per second) of the impacts would rapidly diminish with time. But the pressure of a gas is an increasing function of

both these factors, so it too would tend to zero: a balloon filled with gas at constant temperature would soon go flat on its own.

We must therefore accept that molecular impacts on the container walls are elastic; the same conclusion extends to any collisions between molecules, although, as noted above, these can be neglected in very rarefied gases. The elasticity of collisions implies that molecules undergo no permanent deformation as a result of a collision, consistent with the hypothesis that molecules are structureless: in effect, we treat them as perfectly rigid, extremely small spheres.

The ideal gas equation of state,

$$PV = nRT$$

where  $n$  is the number of moles and  $R$  is the ideal gas constant implies the *Boyle's Law*, according to which at constant temperature, pressure and volume are inversely proportional. Boyle's law aligns very well with experimental data when the gas is very hot and rarefied—conditions that correspond to the kinetic molecular theory of gases. Therefore, the theory can be considered confirmed if, based on its hypotheses, it is found that in isotherms, pressure and volume are inversely proportional. To apply the equation to calculate one of the variables in terms of the other three, the value of the constant  $R$  must be measured; experimentally, it is found that for all ideal gases, regardless of their chemical composition,

$$R = 0.082057 \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}} = 8.3145 \frac{\text{J}}{\text{mol}\cdot\text{K}}$$

## Calculation of the pressure

Let us calculate the pressure  $P$  exerted by an ideal gas. In principle the shape of the container should not matter, since pressure is expected to depend only on the mass and velocity of the molecules; however, for the calculation it is convenient to consider a rectangular box with faces perpendicular to the  $x$ ,  $y$  and  $z$  axes. In particular, let  $d$  denote the distance between the two faces perpendicular to the  $x$ -axis, and  $S$  the area of each of those faces.

Since pressure is the ratio between the normal force acting on a surface and the area of that surface, we need to calculate the force acting on the walls of the container — specifically, on a face perpendicular to the  $x$ -axis. If  $N$  is the number of molecules, the total force  $F$  (the sum of the collision forces exerted by all the molecules) is given by

$$F = Nf$$

where  $f$  is the magnitude of the force exerted by a single molecule during a collision. Since the force of a collision depends on the velocity of the individual molecule, which is not the same for every molecule, it is more convenient to replace  $f$  with its average value over all the molecules,  $\langle f \rangle$ , and to treat  $F$  as a total average force. Because of the very large number of particles, its value is constant over time as long as the thermodynamic variables do not change. So we can write

$$F = N\langle f \rangle$$

We must also take into account that molecules have different velocities. Since velocity is a vector, if  $v_x$ ,  $v_y$  and  $v_z$  denote the three components of  $\mathbf{v}$ , then by Pythagoras' theorem in three dimensions,  $v^2 = v_x^2 + v_y^2 + v_z^2$ . We are interested in average values taken over all the molecules; since chaotic motion has no privileged direction, the mean values of  $v_x$ ,  $v_y$  and  $v_z$  are equal across the whole set of molecules, so  $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$ , where  $\langle X \rangle$  denotes the mean value of  $X$ . The mean value  $\langle v^2 \rangle$  is then the sum of three equal terms, so

$$\langle v^2 \rangle = 3\langle v_x^2 \rangle$$

Furthermore, to simplify the calculation, let us assume that all the molecules have the same mass  $m$  (this is a simplification;  $m$  should be regarded as a mean value<sup>2</sup>).

To find  $f$  we apply Newton's second law in the form

$$f = \frac{\Delta q}{\Delta t}$$

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2 Even a chemically pure gas is not, strictly speaking, made of molecules of identical mass, since most elements occur as a mixture of isotopes.

where  $\Delta q$  is the change in linear momentum of a single molecule due to one collision. Here we use the hypothesis that collisions are elastic. Let us treat molecules as small rigid spheres which, on touching a wall of the container, experience a force  $f$  normal to the wall and directed inward. Kinetic energy is conserved, and the magnitudes of  $v_x$ ,  $v_y$  and  $v_z$  do not change. Since the force is normal to the surface struck, only  $v_x$  reverses sign. The change in momentum  $\Delta q$  therefore equals its only nonzero component,  $\Delta q_x$ :

$$\Delta q = \Delta q_x = m v_{x2} - m v_{x1}$$

where  $m$  is the mass of the molecule, and the subscripts 1 and 2 refer respectively to the instants just before and just after the collision. Since  $v_{x1} = -v_{x2}$ , taking the magnitude gives

$$\Delta q = 2m v_x$$

To find the force  $f$ , we divide  $\Delta q$  by the time  $\Delta t$  between two successive collisions with the same wall.<sup>3</sup> Time equals distance over speed; here the distance traveled is twice the separation  $d$  between the walls perpendicular to the x-axis — after hitting one wall, a molecule will strike others before returning to the same wall, but the net distance covered along the x-axis is  $2d$  — and the relevant speed is  $v_x$ . So, for a single molecule of velocity component  $v_x$ , we get

$$\Delta t = \frac{2d}{v_x}$$

and

$$f = \frac{2m v_x}{\frac{2d}{v_x}} = \frac{m v_x^2}{d}$$

We now take the mean value  $\langle f \rangle$ , replacing  $v_x^2$  with  $\langle v_x^2 \rangle$ . Multiplying  $\langle f \rangle$  by the number  $N$  of molecules gives the total force  $F$  exerted by the ideal gas on a wall:

$$F = \frac{Nm \langle v_x^2 \rangle}{d}$$

To find the pressure  $P$ , we divide  $F$  by the area  $S$  of the wall, so that  $d \cdot S = V$  (the volume of the container) appears in the denominator. The same result is obtained by considering the other faces

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3 The impact force exerted on the molecules during a collision is far greater than that calculated based on the time of flight between two consecutive collisions with the same wall; however, when calculating the average intensity of the collisions, one must account for the time interval between consecutive impacts, during which the force acting on the molecule is zero.

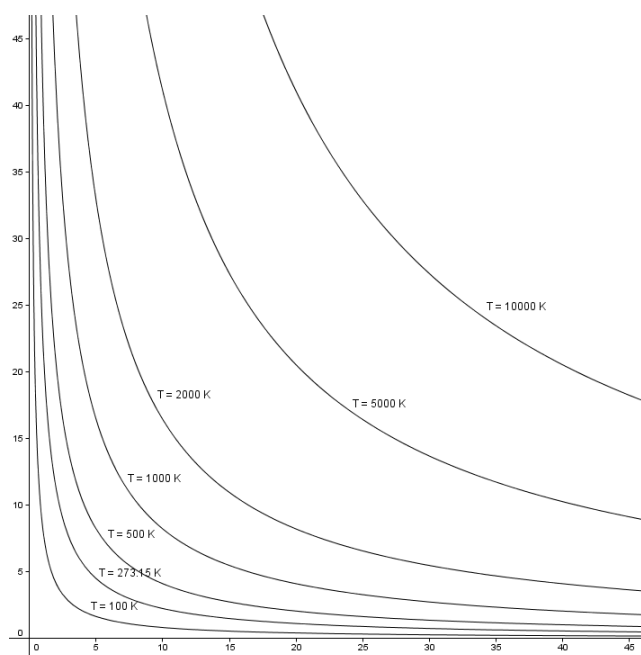
and the y and z axes: pressure does not depend on which axis is chosen for the calculation, as one would expect, since pressure must be uniform in every direction. Hence

$$P = \frac{Nm\langle v_i^2 \rangle}{V}$$

where the index  $i$  can be x, y or z. Since  $\langle v^2 \rangle = 3\langle v_i^2 \rangle$ , we finally obtain

$$P = \frac{Nm\langle v^2 \rangle}{3V}$$

This formula involves purely mechanical quantities ( $m$  and  $v$ ) and has a statistical character, since it contains the average value of the square of the molecular velocity.



The graph of pressure (in atm) versus volume (in liters) according to the ideal gas equation of state for one mole of gas. The curves correspond to different values of absolute temperature.

## Absolute temperature is a measure of molecular energy

We can now turn to the meaning of absolute temperature according to kinetic theory. From the state equation of an ideal gas,

$$PV = nRT$$

where  $n$  is the number of moles and  $R$  is the ideal gas constant, we can express  $P$  as

$$P = \frac{nRT}{V}$$

Equating the two expressions for the pressure gives

$$\frac{Nm\langle v^2 \rangle}{3V} = \frac{nRT}{V}$$

The total number of molecules  $N$  equals the number of moles  $n$  times Avogadro's number<sup>4</sup>  $N_A$  (the number of molecules in one mole); we then obtain the following mechanical and statistical expression for absolute temperature:

$$T = \frac{N_A m \langle v^2 \rangle}{3R}$$

This formula confirms the founding idea of the whole theory: absolute temperature is a statistical measure of the chaotic motion of the molecules.

To generalize this formula to a non-homogeneous mixture of gases,  $m\langle v^2 \rangle$  must be replaced by  $\langle mv^2 \rangle$ .

It is useful to relate temperature to the average kinetic energy  $\langle E_c \rangle$  of a molecule. Since  $\langle mv^2 \rangle = 2\langle E_c \rangle$ , we get

$$T = \frac{2\langle E_c \rangle N_A}{3R} \Rightarrow \langle E_c \rangle = \frac{3RT}{2N_A}$$

The ratio between the gas constant  $R$  and Avogadro's number  $N_A$  is the Boltzmann constant,  $k$ , a very important quantity in thermodynamics, equal to  $1.380649 \times 10^{-23}$  J/K (exact value, SI 2019).

This gives, finally,

$$\langle E_c \rangle = \frac{3}{2} k T$$

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<sup>4</sup> The number of molecules in one mole is always Avogadro's number (or Avogadro's constant), about  $6.02 \times 10^{23}$  for any substance. This is why the mole — not mass — is used as the unit for the “amount of substance”.

So, according to the simplest version of kinetic theory, the average kinetic energy of a molecule of an ideal gas is proportional to the absolute temperature of the gas. This relation also holds for gas mixtures.

Although it is not made explicit in the formulas, the velocity and mean-square velocity above are always measured relative to the container, which is assumed to be at rest. The uniform translational motion of the container as a whole does not affect temperature, which depends only on the random component of the motion. The same principle applies to liquids: an organized motion such as a uniform rotation does not, by itself, raise the temperature.

The relationship between kinetic energy and absolute temperature derived in the kinetic molecular theory extends to all phases of matter, at least in classical mechanics, on the basis of the equipartition theorem, according to which the average kinetic energy is distributed uniformly over all the degrees of freedom of a physical system.

However, in the other phases of matter and in real gases, the total energy also includes potential energy, which generally makes the relationship between total energy and absolute temperature more complicated.

## Polyatomic gases

The relation  $\langle E_c \rangle = (3/2) k T$  holds only for a perfect gas in which rotations and vibrations of the molecules are ignored.

In reality, molecules are often made up of two or more atoms, so these additional motions must also be taken into account. Since ignoring rotations and vibrations means considering only translational motion, the equation above actually gives the mean translational kinetic energy alone — which coincides with the total kinetic energy only for a monatomic molecule. To extend the theory to a polyatomic ideal gas, we apply the principle of equipartition of energy, according to which the average energy is distributed evenly among all the degrees of freedom of the molecule — the independent coordinates needed to specify the position and orientation of the molecule. For a point particle the degrees of freedom are simply its position coordinates, e.g. the three Cartesian coordinates  $x, y, z$ . The mean kinetic energy associated with a single degree of freedom is therefore

$$\langle E_{c,1} \rangle = \frac{1}{2} k T$$

which can be regarded as the fundamental relation between energy and temperature.

A diatomic molecule has five degrees of freedom (three translational and two rotational): if the chemical bond is treated as rigid, one atom can be thought of as orbiting the other, and its relative position is fixed by two angles,  $\theta$  and  $\phi$ . For a nonlinear molecule with three or more atoms there are three rotational degrees of freedom — for example, a third atom can rotate about the axis joining the first two, at a fixed distance from each; in this case, the molecule has a total of 6 degrees of freedom (Note that this picture assumes a bent, nonlinear geometry: a linear triatomic molecule, such as  $\text{CO}_2$ , behaves rotationally like a diatomic molecule and has only two rotational degrees of freedom, not three). In principle, the vibrational degrees of freedom should also be added, which are negligible at room temperature (300 K) and up to temperatures of hundreds or thousands of degrees, depending on the case. Ignoring vibrational degrees of freedom, the mean kinetic energies of a diatomic and of a (nonlinear) triatomic molecule are, respectively,

$$\frac{5}{2} k T \quad \text{and} \quad 3 k T$$

## Results of kinetic theory

### *Real gases*

Kinetic theory can be extended to real gases, liquids and solids by allowing for attractive and repulsive forces between molecules. For a real gas, the covolume — the total volume occupied by all the molecules — must also be taken into account, so the ideal-gas state equation  $PV = nRT$  has to be replaced by a more complex equation, such as the Van der Waals equation.<sup>5</sup>

### *Solids*

In solids, molecules are assumed to oscillate about fixed equilibrium positions, so the mean total energy of a molecule is the sum of its potential and kinetic energy. Assuming that, on average, potential energy equals kinetic energy, the total energy of a molecule in the solid state is twice its kinetic energy. For monatomic solids such as metals, the average total energy of a molecule is

$$\langle E \rangle = 3kT$$

### *Root-mean-square velocity of the molecules*

Consider a physically homogeneous perfect gas (i.e. one in which all molecules have the same mass). We can calculate the root-mean-square velocity ( $v_{\text{rms}}$ ) of the molecules — the square root of the mean square of their translational velocity:

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_N^2}{N}}$$

where  $N$  is the number of molecules and  $v_i$  the translational velocity of the  $i$ -th molecule. Starting from

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

we obtain

$$\langle v^2 \rangle = \frac{3kT}{m} \Rightarrow v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

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<sup>5</sup> According to the Van der Waals equation, the pressure is given by

$$P = nRT / (V - nb) - a n^2 / V^2$$

where  $n$  is the number of moles and  $a$  and  $b$  are positive constants characteristic of each gas; in particular,  $b$  is the covolume of one mole.

From this microscopic formula we can obtain an expression in macroscopic quantities by multiplying numerator and denominator by the number of molecules  $N$ . Since  $k N = n(k N_A) = n R$  (where  $n$  is the number of moles) and  $m N = n(m N_A) = n M$  (where  $M$  is the molar mass), we get

$$v_{\text{rms}} = \sqrt{\frac{3 R T}{M}}$$

or, since for one mole  $RT = PV$  and  $V/M = 1/\rho$ , where  $\rho$  is the density,

$$v_{\text{rms}} = \sqrt{\frac{3 P}{\rho}}$$

### *Graham's law of diffusion*

According to this experimental law, the average velocities of the molecules of two gases at the same pressure are inversely proportional to the square root of their respective molar masses:

$$\frac{v_a}{v_b} = \sqrt{\frac{M_b}{M_a}}$$

where  $a$  and  $b$  label the two gases. This means that, at equal temperature, lighter gases diffuse faster.

This law follows from the root-mean-square velocity formula derived above, since statistics shows that  $v_{\text{rms}} = \sqrt{\frac{3\pi}{8}} v_m$ , where  $v_m$  is the average speed. The ratios between root-mean-square velocities and between average velocities are therefore equal. Assuming equal temperatures,

$$\frac{v_a}{v_b} = \sqrt{\frac{v_{\text{rms},a}}{v_{\text{rms},b}}} = \sqrt{\frac{3RT/M_a}{3RT/M_b}} \text{ ecc.}$$

### *Dalton's law of partial pressures*

This law states that the total pressure of a mixture of ideal gases equals the sum of the partial pressures exerted by each (chemically homogeneous) gas. Rewriting

$$P = \frac{N m \langle v^2 \rangle}{3 V}$$

in the form

$$P = \frac{2 N \langle E_C \rangle}{3 V} = \frac{N \langle m v^2 \rangle}{3 V}$$

where  $N$  is the number of all the molecules and  $\langle E_c \rangle$  their average kinetic energy, we obtain a formula valid also for a mixture with  $N$  molecules.

If the gas labeled “k” contains  $N_k$  molecules of mass  $m_k$  and mean-square velocity  $\langle v_k^2 \rangle$ , we can write

$$\langle m v^2 \rangle = \frac{\sum_k N_k m_k \langle v_k^2 \rangle}{N} \Rightarrow P = \frac{\sum_k N_k m_k \langle v_k^2 \rangle}{3V} = \sum_k P_k$$

which is precisely Dalton’s law.

The law can also be obtained directly from the state equation: for each gas in the mixture  $P_k = n_k RT / V$ . In fact, ideal gas molecules do not interact with other molecules. Then

$$\sum_k P_k = \frac{(\sum_k n_k) RT}{V}$$

and since  $\sum_k n_k = n$ , we get

$$\sum_k P_k = \frac{nRT}{V} = P.$$

### *Internal energy of an ideal gas*

It is well known that the internal energy  $U$  of an ideal gas depends only on its temperature. Kinetic theory explains exactly why: the internal energy of a perfect gas equals the total kinetic energy of its molecules, so

$$U = N \langle E_c \rangle = \frac{1}{2} N f k T$$

where  $f$  is the number of degrees of freedom and  $k$  the Boltzmann constant. Since  $N = n N_A$  and  $N_A k = R$ , we obtain

$$U = \frac{1}{2} f R T ; \quad \langle E_c \rangle = \frac{1}{2} f k T$$

### *Internal energy of a monatomic solid (metals, etc.)*

By the same method used for an ideal gas, the internal energy of one mole of a monatomic solid substance is

$$U = 3RT$$

(recall that the particles of a solid oscillate about their equilibrium positions, so their total energy is the sum of kinetic and potential energy, which are equal on average over time).

### *Molar heat capacities*

The molar heat capacity  $C_M$  of a substance is defined as the heat capacity of one mole of that substance:

$$\Delta Q = n C_M \Delta T$$

where  $\Delta Q$  is the heat transferred to one mole of a substance that is not undergoing a phase transition. For an ideal gas at constant volume,  $\Delta Q = \Delta U$ , since all the heat absorbed by the gas is converted into internal energy. Denoting the molar heat capacity at constant volume by  $C_v$ , we get

$$C_v \Delta T = \Delta U = \frac{1}{2} f R \Delta T$$

and finally

$$C_v = \frac{1}{2} f R$$

where  $f$  is the number of degrees of freedom of the molecules (3 for a monatomic gas, 5 for a diatomic one, 6 for a triatomic or polyatomic one).

The molar heat capacity at constant pressure,  $C_p$ , is obtained from the first law in differential form,  $dQ = dU + PdV$ , by dividing all terms by  $dT$ ; since for one mole  $dU = C_v dT$  and  $PdV = R dT$ , we obtain

$$C_p = C_v + R$$

or

$$C_p = \frac{1}{2} f R + R = \frac{f+2}{2} R$$

from which we derive

$$\frac{C_p}{C_v} = \frac{f+2}{f}$$

For monatomic solids (metals, etc.), the molar heat capacity is given by

$$C_M = 3R$$

( $3R \approx 6 \text{ cal}/(\text{mol} \cdot \text{K})$ ), in agreement with the experimental Dulong–Petit law (which holds only above a critical temperature that depends on the chemical composition of the substance).

### *Calculation of the exponent in adiabatic transformations*

The equation of an adiabatic process for a given quantity of ideal gas is

$$PV^\gamma = K$$

where  $\gamma$  and  $K$  depend on the molecular structure of the gas.

From the first law in differential form, setting  $dQ = 0$ , we get

$$dU + P dV = 0 \Rightarrow \frac{1}{2}fRdT + RT\frac{dV}{V} = 0 \Rightarrow \frac{dT}{T} = -\frac{2}{f}\frac{dV}{V} \Rightarrow T = CV^{-2/f}$$

where  $C$  is a constant dependent of  $f$ . Substituting into the state equation  $PV = nRT$  gives

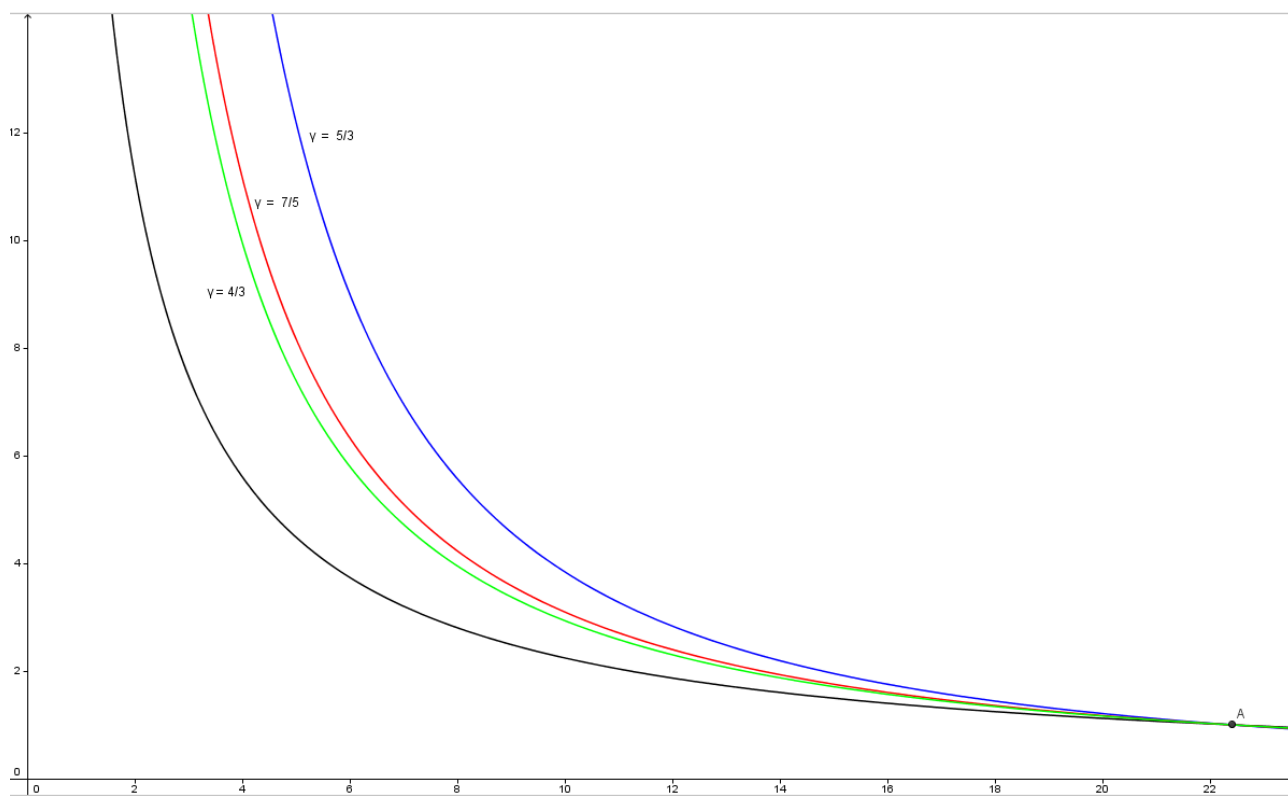
$$PV = nRCV^{-2/f} \Rightarrow PV^{1+2/f} = nRC = K \Rightarrow PV^{(f+2)/f} = K$$

from which we deduce  $\gamma = (f+2)/f$ , or

$$\gamma = \frac{C_p}{C_v}$$

To apply the adiabatic equations, it is necessary to calculate the integration constants  $C$ , given by  $C = PV^\gamma/nRT$ ; setting  $n = 1$  under standard conditions (1 atm, 1 L, 273.15 K) yields the following values.

| $f$ | $\gamma$ | $C$  |
|-----|----------|------|
| 3   | 5/3      | 7.95 |
| 5   | 7/5      | 3.47 |
| 6   | 4/3      | 2.82 |



The colored curves are the adiabats corresponding to  $\gamma = 5/3$  (monatomic gases),  $7/5$  (diatomic), and  $4/3$  (polyatomic). The isotherm is drawn in black. The curves intersect at the point A corresponding to standard conditions (STP) for one mole.

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