

Space, Time, and Motion in Classical Physics

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Introduction

Space, time, and motion are the foundational elements of any physical description of the world that seeks to maintain a continuity with common experience. Even while acknowledging the provisional and illusory nature of that immediate perception of the phenomena we observe in everyday life, it is impossible to approach physics without first describing the world as composed of objects moving through space as time passes. Historically, the emergence of modern mechanics in the seventeenth century and its subsequent formal developments in the eighteenth brought these premises to their extreme conclusions, and already during the nineteenth century those problems emerged which lay at the root of the crisis of the twentieth century, giving rise to the two theories of Relativity and Quantum Mechanics. In truth, today we cannot claim to have reached the end of this entire journey, since a "final" theory of synthesis (should it ever be experimentally confirmed) will need to reconcile two views that are difficult to bring into harmony: the deterministic and reversible one of Relativity, and the probabilistic and non-local one of Quantum Mechanics.

The author believes it is impossible to fully understand all the reasons why the development of the physical sciences in the West assumed that radical and irreversible character which transformed the world, giving rise to industrial civilization and so-called modern culture. Various explanations may be offered, of a social and political nature; moreover, such analyses depend to a considerable extent on the type of culture and the ideological orientation of those who undertake them. One must not underestimate the importance of mathematics, already developed by the Greeks in the forms of geometry and refined at the dawn of the modern era by the developments of algebra, infinitesimal calculus, and so forth, as well as the systematization of deductive thought carried out by Aristotle. Without the power of numerical and symbolic computation and the capacity to handle extremely sophisticated calculation techniques, physics would be impossible. Mere curiosity toward natural phenomena can hardly account for the results achieved by Kepler, Galileo, Newton, and other less well-known figures. The progress of techniques played an essential role, but it was made possible to a great extent thanks to the developments of mathematics.

But this implies that the physical scientist transfers into the interpretation of the phenomenal world the mathematical method and the ideas of geometry to such a degree as to make them predominant. Classical physics is founded on Euclidean geometry, transforming the geometric point, ideally devoid of extension, into the material point, an ideal particle that moves along trajectories analyzable mathematically by means of differential calculus. This geometric foundation of classical mechanics is evident above all in the work of Galileo and Newton, and it was the paradigm of physics until the early twentieth century, even though it had already been understood for decades that there is no single possible geometry capable of describing reality, but that all coherent geometries are equally "true"—that is, those which satisfy the principle of non-contradiction, the sole permanent presupposition of every mathematical system and of every consistent physical description of the world.

The problems pertaining to the nature of space and time have been the subject of investigations spanning diverse fields, not limited to physics alone. In this essay, we shall address only those questions that can be explored and potentially resolved through the observation of phenomena and by means of the tools offered by mathematics. Nevertheless, the objects of this inquiry concern the conditions of our experience, and re-

flection on space, time, and motion is not exhausted by experimental information or calculations alone. The conclusions reached by physicists do not depend solely on observations, formulas, geometry, and the like; the very origin of these problems, the attempts to resolve them, and the proposals put forward all belong to that branch of philosophy which is oriented toward the exploration of nature. Both the physicists who reflected on the nature of space and motion, and the mathematicians who developed non-Euclidean geometries, were, in their own way, philosophers. The exploration of certain problems, so long as no widely shared solution exists, or so long as no method for tackling them is known, is entrusted to reasoning, to the comparison of different positions, and to the capacity to formulate hypotheses and deduce their consequences. Galileo's great discoveries were not based solely on calculations; his *Dialogue Concerning the Two Chief World Systems* contains very few of them; rather, it is a continuous succession of hypotheses, theses, and counter-theses. With Newton, the consequences of general principles are treated mathematically, but until the early twentieth century, the philosophical and conceptual approach to the problem of space and motion remained intertwined with the progress of mathematics. This holds true as well for the relativity of simultaneity, for the measurement of time and lengths, and ultimately for the Equivalence Principle—the bridge from the classical conception, in which space and time are separate and independent of the observer, to the present-day conception of General Relativity and cosmology.

Einstein himself revealed that intuition, or more precisely insight, was at the origin of his ideas.

It has not been possible for me to present this subject in a more concise manner without sacrificing the clarification of certain essential points. School presentations, for reasons of curricular organization, and above all to avoid revisiting questions that modern operational treatments simply bypass, do not go beyond a certain point in exploring the subject at hand; yet in classical dynamics, the problem of the nature of space and motion has never been resolved. This becomes clearer if one properly understands the meaning of Relativity and the principle of identity between inertial and gravitational mass. A satisfactory description, both conceptually and experimentally, of the fundamental concepts of mechanics requires the Einsteinian synthesis; without it, mass, energy, space, and time remain separate and are not clearly defined: they are merely concepts presumed to be known, without one's understanding their profound nature.

Almost the entirety of this essay is devoted to the nature of space and motion, through an exploration of the evolution of these concepts over the centuries. Time presents greater difficulties, to which I have alluded in the chapter devoted to it. It seems that before Einstein, no one ever thought that the measurement of time could depend on the observer. Not only that, but ideas on this subject vary from culture to culture and have little scientific relevance. In practice, time has been treated as a continuous succession, assuming—in part contrary to common experience, as psychology reveals—that it "flows" uniformly, and so forth. The ratio between the length of the portion of this essay devoted to space and that reserved for time reflects a historical reality: it is probably smaller than the ratio between the number of scientific studies on space and those on time published from the second half of the seventeenth century up to Einstein.

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1. Reference Frames and Absolute Space

The first of Newton's laws of dynamics is the principle of inertia. The statement, as formulated by Newton himself in the *Principia Mathematica*, is as follows: "*Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.*"¹ This formulation, taken in isolation, is manifestly incomplete. Relative to whom or to what can a motion be defined as rectilinear and uniform? When we observe or describe the motion of a body (that is, when we evaluate its velocity and acceleration), we do so with reference to some body or environment, or even with respect to ourselves; for example, if we study the oscillations of a pendulum in a laboratory, we assume that the laboratory and we ourselves are at rest, and we refer the motion of the pendulum to the vertical line passing through the point of suspension; in doing so, we neglect the Earth's rotational motion, the Earth's revolution around the Sun, and so forth; the description we obtain is therefore relative to a given observer, and it would be quite different if it were referred to the center of the Earth or to some other body in the Universe. Are all possible ways of describing motion equally arbitrary? Or does there exist a privileged point of view, corresponding to the only correct description, while all the others are, in a sense, merely apparent?

Newton had certainly explored these problems, but he thought he could anticipate objections to his ideas by preceding the formulation of the laws of dynamics with certain principles he considered unassailable, so that their consequences would be unassailable as well. His procedure was the axiomatic one of geometry, from which he derived, in a sense, an idea of time and space as self-subsistent entities, distinguishing "true" time, space, and motion, insofar as they are independent of observation and events, from their apparent, relative, observation-dependent counterparts. Newton's absolute space is akin to Euclidean space, in that the latter contains within itself figures that can be transformed, translated, and so on, while it itself, unlike these figures, is immutable. In the same way, material bodies can be deformed, displaced, and so forth, but space always remains the same. As for time, Newton conceives of absolute time not as that marked by our physical clocks or by astronomical cycles (which present inevitable irregularities), but as that which would be measured by an ideal and perfect clock, whose cycles succeed one another with a rigorously uniform rhythm, identical for everyone and everywhere.² But he sensed that there was some hidden difficulty in his exposition, and the distinction he drew between absolute and relative or apparent space, time, and motion seems to be highly functional in countering the unsettling questions that his own exposition could give rise to.

In his axiomatic formulation, Newton took care to avoid investigations that might have weakened his construction of dynamics; in the *Principia Mathematica*, he explicitly stated that he did not wish to define space, time, place, and motion precisely because he considered them concepts already known to all. Immediately after defining certain less common technical terms, Newton writes in the *Scholium* (the commentary that follows the initial definitions): "*Thus far we have defined terms less commonly known. I do not define time, space, place, and motion, as being well known to all.*" This position is reinforced by the following statement: "*Moreover, the vulgar conceive of these quantities in no other way than in relation to sensible objects.*" In this, Newton invokes the claim that ordinary sensibility does not grasp the essence of things, an idea perfectly compatible with the *forma mentis* of the mathematician, who neglects the form of the concrete object in order to devote himself to the study of abstract figures; but he must have realized that, had he not placed

1 "Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum suum mutare."

2 Newton, in the *Scholium*, explicitly complains of the fact that there is no perfect physical clock in nature. He gives the example of solar days, which are not all exactly equal. Therefore, for Newton, the time of real clocks belongs to the category of "apparent and common" time.

non-empirical principles at the beginning of his exposition, he would have had to face considerable difficulties, such as defining a universal criterion for establishing whether a body is in motion or not, or determining empirically whether a motion is uniform or not in the absence of any reference to material bodies. He probably had in mind the conflict between the heliocentric and geocentric systems: the latter was better suited to describing appearances, that is, the motions of celestial bodies as they are observed from the Earth, but it is erroneous, since in reality the planets revolve around the Sun.

Employing modern terminology, we can state that according to Newton there exists a privileged reference frame, which is not an object or a system of material bodies and therefore cannot be identified through observation; motion with respect to individual material bodies (including observers) is always relative; the privileged system is space itself, and therefore a motion can be called rectilinear and uniform (or, conversely, accelerated) only in reference to space as he conceived it. If matter were removed, space would continue to exist unmodified. Such space is defined by Newton as absolute (that is, independent of the observer and of the bodies contained within it), true (insofar as it is opposed to the spaces or environments to which we in fact generally refer, which are "apparent"), and mathematical (insofar as its structure is defined by Euclidean geometry, the only one considered true in his time); briefly, it is called absolute space.

To be fair, when one attributes to Newton the paternity of the concept of absolute space, one does a disservice to a school of thought already well established; let us allow the philosopher and future bishop George Berkeley to speak in his short treatise *De motu* (1721). "*Peripatetici pro varietate mutationum, quas res aliqua subire potest, varia motus genera distinguebant. Hodie de motu agentes intellegunt solummodo motum localem. Motus autem localis intelligi nequit nisi simul intelligatur quid sit locus; is vero a neotericis definitur pars spatii quam corpus occupat, unde dividitur in relativum & absolutum pro ratione spatii. Distinguunt enim inter spatium absolutum sive verum, ac relativum sive apparens. Volunt scilicet dari spatium? Spatium undequaque immensum, immobile, insensibile, corpora universa permeans & continens, quod vocant spatium absolutum. Spatium, autem, a corporibus comprehensum, vel definitum, sensibusque adeo subjectum, dicitur spatium relativum, apparens, vulgare.*"³ Not only the meaning, but also the lexicon is that of Newton, who had followed the teachings of his predecessor at Cambridge, Isaac Barrow, and shared the ideas current in the learned circles of his era.

Berkeley's critique is primarily of a philosophical-psychological nature, rather than physical. Indeed, immediately afterward, in section 53, our author continues as follows: "*Fingamus itaque corpora cuncta destrui & in nihilum redigi. Quod reliquum est vocant spatium absolutum, omni relatione quae a situ & distantis corporum oriebatur, simul cum ipsis corporibus, sublata. Porro spatium illud est infinitum, immobile, indivisibile, insensibile, sine relatione & sine distinctione. Hoc est, omnia ejus attributa sunt privativa vel negativa. Videtur igitur esse merum nihil. Parit solummodo difficultatem aliquam quod extensum sit. Extensio autem est qualitas positiva. Verum qualis tandem extensio est illa, quae nec dividi potest, nec mensurari, cujus nullam partem, nec sensu percipere, nec imaginatione depingere possumus? Etenim nihil in imaginationem cadit, quod, ex natura rei, non possibile est ut sensu percipiatur, siquidem*

3 "The Peripatetics, according to the variety of changes which a thing could undergo, distinguished various kinds of motion. Today, on the contrary, those who study motion understand only local motion. But local motion cannot be understood unless we simultaneously understand what place is; and this, according to the moderns, is defined as the part of space which a body occupies, whence it is divided into relative and absolute according to the nature of the space. For they distinguish between absolute or true space, and relative or apparent space. They wish to say, that is, that there is given a space? A space everywhere immense, immovable, insensible, pervading and containing all bodies, which they call absolute space. Space, on the other hand, comprehended or defined by bodies and thus subject to the senses, is called relative, apparent, common space." *De motu*, pp. 18–19.

imaginatio nihil aliud est quam facultas representatrix rerum sensibilibus, vel actu existentium, vel saltem possibilium."⁴

However, the analysis of the forces acting in circular motion that he proposed is not satisfactory, and it seems that Berkeley did not fully grasp the meaning of the experiment of the water contained in a bucket set in rotation about its axis. The centrifugal forces that raise the liquid toward the outside manifest themselves only when the liquid is animated by a circular motion relative to the laboratory environment, that is, relative to the Earth—this is the result of the experiment, because it is what the experimenter observes independently of how the phenomenon is interpreted. These centrifugal forces are not at all relative to some observer or other body, inasmuch as they produce effects that depend neither on the point of observation nor on the presence of the experimenter or of other objects in the vicinity. One might object that in reality the observed effects depend on rotation relative to the Earth itself, or to the fixed stars, or to any system of bodies considered to be at rest with respect to space, and therefore there would be no need for an absolute space to which rotary motion must be referred; but this is an objection of a different kind, one that is not explicitly formulated by Berkeley. Moreover, it is difficult to think that the Earth or other bodies act upon a rotating mass in such a way as to create centrifugal forces, whose intensity depends on the distance from the axis of rotation and on the velocity of rotation. The simplest explanation was precisely the Newtonian one: these forces originated from the motion of the rotating mass, not from the presence of other bodies, and this motion, given that the effects were objective and not relative, must itself be objective, that is, absolute according to the lexicon of the era.

Berkeley's criticisms, whether because Newton's dynamics proved so effective as to explain many phenomena that had until then been disconnected from one another and poorly understood, or because they were not sufficiently well formulated within the framework of the physical sciences, did not gain acceptance in subsequent epochs, until Mach developed a far more effective and radical critique of the idea of absolute space.

4 "Let us therefore imagine that all bodies are destroyed and reduced to nothing. What remains is called absolute space, with every relation that arose from the position and distances of bodies removed along with the bodies themselves. Moreover, that space is infinite, immovable, indivisible, insensible, without relation and without distinction. That is to say, all its attributes are privative or negative. It therefore seems to be mere nothing. Only the fact that it is extended creates some difficulty. Extension, however, is a positive quality. But what kind of extension is that which can be neither divided nor measured, of which we can perceive no part with the senses nor depict with the imagination? For nothing enters the imagination that, by the nature of the thing, cannot possibly be perceived by the senses, since the imagination is nothing other than the faculty of representing sensible things, whether actually existing or at least possible."

2. The Principle of Classical Relativity

At this point, two problems arise: 1. How can Newton's theory of space be demonstrated—or at least justified? 2. Assuming as a hypothesis that Newton's ideas are correct, how can we establish the "state of motion" of an observer or of a body with respect to that space?

A thorough analysis of the second point leads (according to Newton) to the justification of the first; therefore, it is appropriate to examine the state of motion of a material object with respect to space.

First of all, one may observe that this problem is by no means idle or negligible. It may seem, indeed, that establishing a relation between bodies and space is useless: why not consider all motions as relative to material objects, and accept all possible descriptions of a motion in general as equally valid, save for selecting, case by case, the most suitable one? After all, this is what is done in practice: when we state that the velocity of, say, an automobile is 50 km/h, we refer to the Earth's surface, without asking ourselves what the "real" velocity might be with respect to some hypothetical "absolute space" that would be entirely superfluous in that context. In reality, the history of astronomy seems to contradict such a thesis⁵: if all motions are considered relative and the choice of a body or reference frame is arbitrary in principle, why should we consider the planets as revolving around the Sun and not around the Earth? What sense could the comparison between the Ptolemaic and Copernican systems have had, or indeed the very proposal of the latter as the true system of the world? And further: why can we affirm that the Earth rotates about its North-South axis in 24 hours? With respect to what do the planets, the Earth, and any body in motion move? Evidently, the various reference frames do not appear to be equally valid, but constitute a hierarchy. Even the Sun is in motion with respect to the Milky Way, and galaxies are not immobile. Individual bodies or systems of bodies, considered fixed within a local domain, are not so within a more vast space.

The prevalence of the heliocentric hypothesis over the geocentric one demonstrates that some descriptions have been considered "truer" than others. This suggests that privileged reference frames exist, and since these cannot be referred to a given body or system of bodies, they may eventually consist in the relation with space itself. Apart from considerations of an astronomical nature—which clearly indicate that every material system we can choose as a reference turns out to be in motion with respect to a larger ensemble, and that none of them appears to be at rest in the Newtonian sense of the term, since we can observe only relative motions—there is nevertheless a fundamental reason in favor of Newton's theory, or of any other theory that, while diverging from it in some respect, preserves a distinction between correct and incorrect descriptions, that is, between "valid" and "non-valid" observations. This reason is the very possibility of physics as a body of theories endowed with objective meaning: the laws of physics must be objective (that is, not dependent on particular circumstances) and therefore not relative to individual observers, but rather valid for all; yet this is possible only if all individual observers adopt a worldview on the basis of which they agree in describing the

5 The very history of astronomy proves that the idea of an absolute reference enjoyed universal consensus well before Newton's formulations. The representation of the cosmos as an ordered succession of spheres presupposes a fixed center, and this alone, together with the unmoved mover, is an absolute reference frame. The Copernican system was revolutionary because it placed the center in the Sun and not in the Earth, but it postulated a center of the universe just as the Ptolemaic system did; and so too Kepler and Galileo did not formulate the hypothesis that there was no central body. The difference between a "cosmos endowed with a center" and "absolute space" is more descriptive-lexical than conceptual; once the conflict between the two systems was resolved in favor of the Copernican one, and the distinction between terrestrial physics and celestial physics was demolished, attention shifted toward space, because the problem of the center was no longer relevant. Even if the abstraction of absolute space is prior to Newton, he highlighted its necessity (in his view) through the analysis of accelerations and forces, proceeding beyond the intuitive idea—already contained in that of a center of the cosmos—of a container of all bodies.

same class of phenomena; and since this does not always happen in practice, it is necessary to establish whether there are observers more reliable than others, what they are, and how they are distinguished from all the rest. Absolute space, which for Newton represented the perfect reference, justified the existence of physics as an objective science, because it was not a particular observer and constituted a universal reference: it was at the very least a sufficient hypothesis, if not a strictly necessary one, to achieve that purpose. Otherwise, there would be as many possible universes as there are observers, and it would be impossible to conceive of a unitary theory of the Universe.

If this way of conceiving the problem is logically impeccable, it entails in itself serious difficulties. The first is obvious: physics acquires an essentially theoretical and ideal character; indeed, its laws are not "true" for everyone, but only from a very particular point of view—that of the observer at rest with respect to absolute space. Let us reflect on the resistance to abandoning the geocentric system, which was directly connected to observations, albeit imperfect and complicated, in favor of the heliocentric one, which appeared hypothetical, abstract, not immediately capable of predicting celestial motions, and moreover not initially supported by decisive observations in its favor.⁶

The idea of an absolute space that contains everything and with respect to which any movement must be referred is even further removed from common experience and from the results of observations and experiments. To the extent that science acquires certainty and universality, it becomes ever more abstract and remote from ordinary experience (because phenomenal reality is necessarily relative to the observer and is not immediately comprehensible starting from general principles). The second difficulty refers back to the preceding problem: to remedy the abstractness, which borders on unreality, we must know our relation with space. If we do not resolve this problem, even if physics existed as a unitary theory, it would be reduced to an abstract description based on unverified hypotheses.

When one speaks of the relation of a body or of an observer with absolute space, one means at least three pieces of information: position, velocity, and acceleration of the body with respect to space. In order to establish such a relation effectively, these three quantities must be knowable through the same experimental procedures, performable by any observer in any place or time. It is clear that the absolute position of a body cannot be defined, first of all for theoretical reasons: space according to Newton is infinite and homogeneous and therefore cannot have a center (this was instead possible in the Aristotelian geocentric system and in the Copernican heliocentric one); all places in space are indistinguishable and therefore equivalent. However, this is a deduction that Newton did not formulate explicitly, even though he maintained the idea that there was a center of the world, which he discussed in Book III.⁷ The problem now is velocity. One observes that a given body has, with respect to a given observer, a velocity v ; what is its real velocity v ? In this case as well, Newton (and not only he, but virtually all physicists from Galileo onward) answers in the negative: it is not possible to determine experimentally the absolute velocity v of a body (even though nothing prevents us from thinking that it has a definite value: but in physics one requires that a quantity be experimentally deter-

6 Not only that; among the consequences of the heliocentric system there is the apparent absence of stellar parallaxes, observable only with modern instruments. The astronomer Tycho Brahe proposed his own mixed system precisely for this reason.

7 Newton argued that if the distribution of mass were finite, the external matter would tend by gravity toward the interior, collapsing into a single spherical mass at the center. But if matter were instead distributed uniformly in an infinite space, it could never coalesce into a single mass, forming instead infinitely many scattered bodies (the Sun and the fixed stars).

minable; the mere hypothesis that it has a definite value is of no interest, if one cannot establish in some way what that value is⁸).

This conclusion, in reality, does not go back to Newton, but was already acquired and generally admitted. Its empirical justification derives from the so-called "Principle of Galilean Relativity" (in the classical, pre-relativistic sense, not to be confused with the theory of Relativity, which nevertheless substantially incorporates it). The statement is as follows: "If a closed system (in the sense that whoever is inside it cannot receive any information from the outside) moves with uniform rectilinear motion with respect to an external body, it is impossible to perform within it any experiments or observations that reveal its state of rest or of motion with respect to that body"; for example, if we find ourselves inside a railway carriage moving with uniform rectilinear motion with respect to the Earth's surface (this means that we do not feel jolts or accelerations), and if no information reaches us from the outside, we cannot know whether we ourselves and the environment in which we find ourselves are in motion or at rest with respect to the Earth's surface. All the more so, we cannot measure our velocity, which therefore cannot be defined except in relation with the external world: indeed, every phenomenon observed inside a system moving with constant velocity occurs as if that system were at rest. Put differently: to establish one's velocity with respect to a body, one must exchange information with that body—for example, by observing it directly. However, this principle presupposes that one can speak of uniform motion in an absolute sense—that is, the difference between uniform and non-uniform motion must be establishable within a closed system, and in Newton's theory this distinction necessarily refers back to an "absolute" space with respect to which a motion is or is not uniform.

If we cannot measure our velocity with respect to objects outside our capacity of observation, all the more so we cannot establish our velocity with respect to absolute space: evidently, this is not an "observable" entity in the sense commonly attributed to that term, and therefore every environment or every body, with respect to it, is "closed" inasmuch as it is impossible to establish experimentally its relation with an entity that is unobservable by nature. In a certain sense, it is a geometric postulate necessary to justify the difference between uniform and non-uniform motions. Therefore, observations performed in environments moving with uniform rectilinear motion with respect to absolute space are all equivalent, and all observers in uniform rectilinear motion with respect to space must find that, under equal conditions, phenomena occur in the same way, that is, according to the same laws, as if the observers were at rest—it does not matter whether they are "truly" at rest; indeed, the concept of "body at rest" is at this point useless.

8 This requirement has been largely abandoned by current advanced theoretical physics.

3. Newton's Justification of Absolute Space

The preceding conclusions do not extend to acceleration. We must bear in mind that the motion of a body—that is, the variation of its position in time with respect to an observer or a reference frame—is completely described by position, velocity, and acceleration, the latter term being rigorously understood to mean not only the increase or decrease of the rapidity of motion, but also the variation of the direction of motion. According to dynamics, it is not necessary to proceed further, because the forces acting on a physical system are proportional to its acceleration, which makes the latter the most important quantity, the one endowed with real physical content, unlike velocity, which has a descriptive character and is relative to the observer. Let us imagine once again that we are inside a railway carriage, which initially has a constant velocity with respect to the Earth's surface, and that we receive no information from the outside, and let us suppose that, starting from a certain instant, it accelerates; is it possible to detect this acceleration from inside the carriage, without any external information, and to measure it? The answer is affirmative: especially if the velocity of the environment increases rapidly enough, the observer himself feels a force pushing him in the direction opposite to that of motion; conversely, the observer feels a push in the same direction as the motion if the environment brakes. In the same way, objects that are not rigidly connected to the environment (that is, that are not constrained to it) appear to set themselves in motion without any force of internal origin acting upon them. If an object is dropped inside a closed system that accelerates along a horizontal plane, the observer inside does not see it fall along the vertical, as would instead happen if the system were not accelerating. The observer who witnesses these phenomena can recognize the acceleration of the environment with which he is connected, at least with respect to the Earth's surface; can he affirm that he is accelerating with respect to absolute space? In this case as well, Newton answers affirmatively: indeed, precisely the observation of these phenomena proves that motion is relative to space.

In reality, Newton claimed to have performed an experiment in support of his theory, that of the rotating bucket, and his argument rested essentially on the analysis of centrifugal forces. In practice, he considered the following experiment. He filled a bucket with water and suspended it with a rope, then twisted the rope several times, until it became rigid (*donec filum a contorsione admodum rigescat*); he waited until the surface of the liquid became flat again, and then let the rope unwind, setting the vessel in rotation. Initially, the motion is transmitted only to the bucket—it takes some time for the water, thanks to contact with its inner walls, to begin to rotate. Therefore, at the beginning of the experiment, there is relative rotation of the water with respect to the bucket, and vice versa. After a while, the water rotates together with the bucket—it is at rest with respect to it—and one observes that the profile of its surface rises toward the edges, to a degree that is greater the faster its rotation. Newton concludes that the centrifugal forces pushing the water particles away from the axis of rotational motion are the manifest consequence of rotary motion, exactly as someone spinning on a carousel feels a centrifugal force. Rotary motion is real to the extent that it produces experimentable and objective effects—were it not for the curving of the water's surface, we could conclude nothing of the kind; every rotation would be relative, just as velocity is. Newton deduces from this that acceleration is determinable in an absolute manner starting from centrifugal forces, and therefore must be referred to an "absolute space" and not to individual material bodies, with respect to which it can only be relative. According to Newton, the manifestation of centrifugal forces in a physical system implies that it is rotating with respect to space. His conclusion mirrors the considerations developed earlier concerning the effects of accelerations of environments that drag into translational motion both the observer and bodies contained within them, whether these are connected to the environment or not rigidly constrained, as in the case of a passenger standing on the floor of a public vehicle. The forces felt by observers and bodies have no

recognizable origin in the action of other bodies, and must be understood as effects of the variation of the environment's velocity with respect to space—an explanation that is even more compelling in the case of centrifugal forces, which diverge from the axis of rotation and have no fixed orientation; there is no way to treat them as forces exerted by other bodies. Let us bear in mind that in Newtonian dynamics, forces are always actions and reactions between material bodies. The so-called forces of inertia, such as those that agitate bodies subject to acceleration, cannot, according to classical dynamics, have their origin in the action of other bodies.

It is appropriate to reiterate, should it not be sufficiently clear from what has been said so far, the difference between the definitions of velocity and acceleration according to kinematics and those according to dynamics. Kinematics is purely descriptive, and its definitions are formal, that is, they do not presuppose the specification of a particular physical reference frame; in practice, one uses Cartesian axes or writes equations without specifying it. Dynamics connects accelerations and forces and requires the choice of a physical reference frame. Without a well-defined theory of dynamics, the problem of space cannot be treated in modern terms; one must confine oneself to philosophical-psychological considerations. But in physical terms, by virtue of Newton's Second Law, the debate on the nature of space depends on the study of accelerations. Newton's rotating bucket experiment falls squarely within this framework. The same holds for Foucault's pendulum, whose plane of oscillation varies in time, revealing the Earth's diurnal rotation; for the direction of the plumb line; and for the eastward deviation of falling bodies. All these observations have in common the fact of being performable inside a closed environment, or at least without having to refer to the fixed stars. This is an absolutely fundamental point, because it clearly separates the measurement of velocity from that of accelerations; while by the Principle of Classical Relativity we cannot establish the state of motion or rest of an observer inside a closed system in uniform rectilinear motion with respect to an external body, we can establish whether it is accelerating or not solely on the basis of observations and experiments performed inside a closed system. Indeed, the strongest evidence for the Earth's diurnal rotation comes from experiments conducted in the laboratory (pendulums, etc.) or on the Earth, neglecting the apparent celestial motions, precisely because they are obtained without considering information coming from outside a laboratory, or from the sky itself.

There remained, however, a problem of no small account, of which Newton showed himself to be fully aware. In fact, we cannot directly obtain information about space and therefore, in principle, we cannot know whether a given motion is correctly described with respect to it; Newton thought he could circumvent, if not resolve, the problem by seeking a stationary reference frame, or at least one that could be considered such with excellent approximation. In practice, his suggestion was to consider all motions as relative to this reference frame. Depending on the case, this may be the Earth for most observations performed on its surface, or the Sun if we wish to evaluate planetary motions, or the fixed stars if we extend our investigations to astronomy. The *stellae fixae* are in fact the ultimate solution; not by chance, the proof of the Earth's annual revolution provided by Bessel more than a century later was based on the observation of the parallax of 61 Cygni.

This transition occurs primarily in Book III of the *Principia*, entitled *De Mundi Systemate* (The System of the World), where Newton moves from abstract mathematics to astronomy. Initially, Newton assumes as an observational datum, among the Phenomena that open Book III, that the fixed stars are at rest relative to one another: from any observation point on Earth, they do not appear to move among themselves, maintaining the same relative positions over the centuries. But Newton does not limit himself to this empirical assumption: in Proposition XIV of Book III⁹ (*Orbium aphelia et nodi quiescunt*), he demonstrates that the aphelia

9 I refer to the 1726 edition.

and nodes of the planetary orbits are at rest, as a consequence of the law of gravitation. In Corollary I to the same proposition, he deduces that the fixed stars are also at rest, since they maintain fixed positions relative to the aphelia and nodes. In Corollary II, he adds an argument of a dynamical nature: the fixed stars, being uniformly distributed in all directions of the sky, mutually cancel out their gravitational attractions (by *Prop. LXX*, Book I), so that none of them tends to fall toward the others. This same reasoning was set forth at length in the letters to Richard Bentley (1693)¹⁰. On the basis of these three conjoined arguments—observational, geometric-demonstrative, and dynamical—Newton concludes that the fixed stars form a rigid, non-rotating framework immersed in absolute space, thus providing the best reference frame accessible to observation.¹¹

In Book III of the *Principia Mathematica*, Newton discusses the position of the center of the world starting from the hypothesis according to which *Centrum systematis mundani quiescere* ("the center of the world is at rest"), concluding *Hinc commune gravitatis centrum terrae, solis & planetarum omnium pro centro mundi habendum est* ("Hence the common center of gravity of the Earth, Sun, and all the planets is to be regarded as the center of the world"). Strictly speaking, this corollary follows from a proposition presented as a hypothesis (*Hypothesis I*), in favor of which he argues as follows: "*Hoc ab omnibus concessum est*," etc. It seems to me that the general sense should be understood in the following way: if we accept that there is a center of the world, and that it is at rest and solidary with the fixed stars, then this is the center of gravity of the solar system.

It is evident that Newton could have explained the effects of rotational motions as relative to the fixed stars without contradicting the results of his experiments on centrifugal forces and without making any reference to absolute space. Indeed, one could argue that it is precisely absolute space, and not the fixed stars, that constitutes the less satisfactory reference of the two: not being a material body, it cannot fit into the scheme of action and reaction upon which Newton himself founds all of dynamics, whereas the fixed stars, however remote, remain at least material bodies in the proper sense. The scientist and philosopher of science Ernst Mach arrived at precisely this conclusion about two centuries later, but in favor of Newton's position there were two factors: the force of a tradition that considered space a necessary element in any physical description of motions, and the fact that from a physical point of view, reference to the fixed stars is no clearer or more comprehensible than reference to space. The fixed stars should be fixed relative to space, but they are not truly at rest; they are animated by local motions, and appear fixed only thanks to their enormous distance from the solar system, which made the measurement of their movements impossible in Newton's time.¹² Their ensemble is not a physical system of solidary bodies, and in astronomy it can be taken as a reference

10 This argument is not correct. The equilibrium of the system of fixed stars would be intrinsically unstable. A perfectly uniform distribution is a mathematical ideal, but physically impossible: the slightest perturbation would trigger the gravitational collapse of matter into clumps. Newton himself, in the letters to Bentley, admitted that such perfect equilibrium could not be the result of natural causes, but required divine intervention.

11 It is important to note that the deduction of the law of universal gravitation does not require, in principle, reference to the fixed stars. The law emerges from the study of the two-body problem, analyzing the relative motion of the planet with respect to the Sun. In an exact calculation, the Sun is not considered a fixed point, but oscillates around the common barycenter of the Sun-planet system. The relative orbit is an ellipse whose dynamics depend on the total mass of the system ($M+m$), not just the mass of the Sun. The barycenter, in the absence of external forces, is at rest or in uniform rectilinear motion, and therefore constitutes a natural inertial reference frame, internal to the system itself. The fixed stars intervene not to deduce the law, but to empirically verify that this barycentric system is indeed at rest with respect to absolute space.

12 In 1718, E. Halley announced that Aldebaran, Arcturus, and Sirius were displaced from their expected positions by substantial fractions of a degree, deducing that each had its own translational velocity across the sky, but the definitive proof of the existence of local stellar motions must be attributed to J. Cassini (1738).

by virtue of an illusion that conceals the depth of the cosmos in order to substitute for it a fixed image on the celestial vault.¹³

Furthermore, until the discovery of the possibility of non-Euclidean geometries, or rather of their relative consistency (if Euclidean geometry is consistent, non-Euclidean geometries must be as well), the only geometry considered true was the Euclidean one. The model of physical space was that of geometric space; but from the moment different geometries became logically possible, there could be no certainty as to which one actually represents space. As early as 1824, Gauss expressed the idea that the decision should be left to empirical observations; the same position was adopted by Lobachevsky; then Riemann, in his famous 1854 habilitation lecture (*On the Hypotheses which Lie at the Foundations of Geometry*), formulated the hypothesis that the geometry of space might not be fixed a priori, but could depend on the distribution of matter within it—an idea that conceptually anticipated General Relativity. The notion of the possibility of many geometries undermined the foundations of the conviction according to which the nature of space, as it had been conceived, was necessary and knowable independently of experience, and that it was independent of the matter contained within it. This same conviction—that space was a necessary structure knowable a priori—had found its most complete philosophical formulation in *I. Kant (Critique of Pure Reason)*, according to whom the representations of space and time are *a priori* forms of sensibility, without which any knowledge of the external world would be impossible. The necessity of Newton's axiomatic construction thus fell away, and the prospect arose of re-examining the problem of motion in a radically different way, possibly re-evaluating relativist positions analogous to that of Berkeley.

13 It should be noted that Newton considered space a sensorium Dei, that is, the organ by which God observed the entire universe. The lexicon is not that of Kant, but the function that the German philosopher attributed to space is in a certain way akin to the Newtonian concept, with the difference that for Kant the *a priori* form is a cognitive category.

4. Inertial and Non-Inertial Observers

Various objections were raised against Newton's theory of space (and of time, which was likewise considered absolute and independent of matter). These can be derived by contradicting the hypotheses—sometimes implicit—upon which he had based his argument. If one analyzes the concept of absolute space, one can conclude that it comprises the following hypotheses: 1. Existence independent of that of matter; 2. Properties (those formulated in Euclidean geometry) independent of the individual observer and of matter; 3. The role of acceleration, which manifests motion with respect to space through centrifugal forces (and, more generally, the forces of inertia). One may therefore object against the priority of space with respect to matter, against the independence of its formal structure (the axioms and theorems of geometry) from matter, and against its very deduction from centrifugal forces.

The most interesting criticisms can be summarized in two counter-theories: 1. The theory of space as a set of relations: in brief, material bodies are a reality prior to space; the latter expresses in a unifying concept the possible relations—compatible with experience, intuition, and logic—that obtain among geometric figures, viewed as ideas expressing properties of bodies such as extension and shape; 2. The theory of science in general as a structure whose elements (concepts and laws) must express experience, and only what can be deduced from it: hence all references to substances, entities, hypotheses, and the like for which no experimental foundation can be found must be eliminated from physics.

The second point of view (which, however, is not incompatible with the first; indeed, the two are often held simultaneously) was upheld against Newton's position especially by the Austrian physicist and philosopher of science E. Mach (1838–1916), whose work *The Science of Mechanics in Its Historical-Critical Development* profoundly influenced the philosophy of science, substantially modifying the approach to the foundations of classical dynamics. According to Mach, Newton's concept of space is not deducible from experience; the distinction between absolute and relative motion (with reference especially to acceleration) would presuppose from the very beginning the idea of an absolute space: if one remains on the ground of facts, one knows nothing but spaces and motions relative to material bodies. The Ptolemaic and Copernican systems are both correct, but the second proved to be simpler and more practical than the first. The Universe is not given to us twice (that is: it is not "factual" to distinguish, for example, between the Earth at rest and the Earth in rotation), but only once, with its relative motions, the only ones that are measurable: on the basis of observation, we can only affirm that the Earth rotates with respect to the fixed stars—every other description makes use of unverifiable hypotheses.

In fact, analyzing Newton's interpretation, one can notice a "logical leap" at the point where he hypothesizes that centrifugal forces manifest motion with respect to absolute space. Let us consider the Earth's rotation: this can be measured in different ways, with operationally independent procedures—for example, by measuring the variation of the plane of oscillation of a pendulum, or the eastward deviation of falling bodies, or also the so-called "complementary" accelerations (which manifest themselves in the formation of cyclonic and anticyclonic currents in the atmosphere: these are phenomena explainable by means of the Earth's rotation). All these observations yield the same result, that is, they can be interpreted by admitting that the Earth rotates in almost 24 hours, without referring to any other body in the Universe. But this period of "absolute" rotation coincides with the

rotation of the sky of fixed stars about the Earth's axis: put differently, the Earth rotates with respect to the fixed stars in precisely almost 24 hours. Could it not be precisely this "relative" rotation that is the cause of the phenomena observed on the planet's surface? Put differently: the cause of centrifugal forces could be (given the close coincidence of the measured rotation times) precisely the rotation relative to the fixed stars, rotation with respect to material bodies, and not a supposed rotation with respect to an ideal space, whose physical reality could not be deduced except through precisely those phenomena for the explanation of which it was introduced. It is not possible to perform an experiment that would decide once and for all which is the correct interpretation, because the theoretical analysis of every such experiment would in fact be founded on some theory of space and motion. Nevertheless, Mach's critique—even if it is not entirely accepted—has led physicists to modify the language in which dynamics is presented, replacing Newton's space with the concept of "inertial observer," I.O.

It is to *Ludwig Lange* (1885) that we owe the exact term "inertial system" (*Inertialsystem* in German¹⁴). Lange defines the inertial system in a purely operational manner: "An inertial system is any reference frame in which a free body (not subject to forces) moves in uniform rectilinear motion." It is essential that an inertial observer—that is, one at rest relative to an inertial system—be free from external forces; we have already seen that, were this not the case, the observer would detect the manifestation of "apparent forces" such as centrifugal forces (for an inertial observer, only the centripetal force that constrains a body to move around a center or an axis is real, not the centrifugal force). Therefore, at least in principle, there exists a practically applicable criterion to decide whether a given observer or reference frame is inertial or not, and there is no need for privileged reference frames deemed to be stationary or nearly so, nor for absolute space itself.

It should be noted that the clarification introduced regarding the definition of an inertial observer, which at first glance may appear redundant, is necessary to operationally establish the condition under which an observer is such. An observer is inertial if and only if the resultant of all external forces acting upon them is zero: this is the same condition satisfied by the center of gravity of a system of bodies at rest, or in uniform translational motion.¹⁵ Without this clarification, the given definition is insufficient: consider a system in free fall and the bodies contained within it, some at rest, others in uniform rectilinear motion relative to the system. An observer at rest relative to this environment detects no violation of the principle of inertia; nevertheless, they are not inertial, being subject to the force of gravity which, within a volume sufficiently small to render tidal forces negligible, imparts the same acceleration to all masses contained within it.

The observer must verify that the objects whose behavior they observe are not subject to external forces, since the motions they measure are always and only relative to their own reference frame, and they cannot proceed to a correct measurement of the forces involved if they themselves are subject to accelerations.

To ensure that a reference frame is inertial, Lange proposed a complicated construction¹⁶, starting from the hypothesis that free bodies exist—that is, bodies not subject to external forces—in order to avoid any refer-

14 The introduction of the term "frame of reference" is due to James Thomson, around those same years.

15 It should be noted that Newton places the center of the world at the barycenter of the solar system. The stars can (and indeed do) have local motions, but the barycenter of the system of fixed stars must remain at rest in order for them to collectively serve as the ultimate reference. In both cases, the center of gravity is the fixed point, not any particular body. The vanishing of the resultant of all forces is essential both for defining fixed points to serve as references and for characterizing inertial observers.

16 See bibliographic reference and description in E. Mach, *The Science of Mechanics: A Critical and Historical Account of Its Development*, trans. T. J. McCormack (Open Court Publishing, 1960, 6th ed.). For a modern overview, see also the entry "Space and Time: Inertial Frames" in the Stanford Encyclopedia of Philosophy.

ence to absolute space. In short, this construction consists in determining three free particles that move in rectilinear motion along three non-coplanar lines, relative to which every other free body moves in rectilinear motion. This result can be refined as follows: relative to a coordinate system in which three free particles move in straight lines and cover distances proportional to one another, the motion of any fourth free particle will be rectilinear and uniform. There is—it seems to me—a difficulty: what assures us that free particles will continue to move in such a way that the ratios between the distances covered remain constant? Evidently, the principle of inertia applied to each particle and the uniform flow of time for all. But then, apart from the ingenious method adopted, one fails to see what the advantage is over the simpler Newtonian formulation. The interest of Lange's proposal, and more generally of the research carried out toward the end of the nineteenth century, consists essentially in exploring the possibility of eliminating "metaphysical" or unobservable entities and their related ideas from physics.

However, one must ask whether Lange's definition constitutes a true elimination of absolute space or merely a reformulation of it in operational terms. If an inertial observer is one relative to whom a free body moves in uniform rectilinear motion, the concept of space inevitably re-enters through the window: to define the "straight line" and the "uniformity" of motion without appealing to an absolute background, one must in fact postulate *a priori* a Euclidean geometry and a universal time in which these ideal motions can occur. Lange's critics, and later Poincaré in his reflection on conventionalism, noted that, since perfectly force-free bodies do not exist in nature due to universal gravitation, the definition of an inertial system nonetheless requires the assumption of a fixed and inert spatial and temporal background. Lange had thus eliminated absolute space as a "substance" (the Newtonian *sensorium Dei*), but he had not succeeded in eliminating its function as an *a priori* geometric structure.

To recapitulate: in Newton's formulation, I.O.s correspond to observers in uniform translational motion relative to space; from a modern point of view, they are the observers for whom the laws of classical dynamics hold true, in the sense that any phenomenon described by an I.O. satisfies Newton's laws. Thus, the motion of a body not subject to forces, measured by an I.O., appears rectilinear and uniform; if an I.O. measures the force $\mathbf{F}$ ¹⁷ acting on a body of mass m , they find that the acceleration $\mathbf{a}$ satisfies the law $\mathbf{F} = m\mathbf{a}$; and so on. Conversely, non-inertial observers perform experiments that contrast with Newton's, and these differences are explained by introducing the concept of "fictitious" or apparent forces—also called "inertial forces."

How can an observer establish their own situation? One can reconsider the example of a railway car accelerating relative to the Earth's surface: anyone inside will observe that all bodies unconstrained to the car (for example, dropped, or simply resting on the floor without being held in any way) will acquire the same acceleration regardless of their respective masses (this independence of $\mathbf{a}$ from m is also characteristic of gravity—a fact which is certainly not coincidental and is susceptible to further analysis¹⁸). It is easy to see that this follows from the principle of inertia, and that this acceleration (relative acceleration, $\mathbf{a}_r$) is equal and opposite to that of the observer measuring it (called the "entrainment acceleration", $\mathbf{a}_i$); that is, one has: $\mathbf{a}_r = -\mathbf{a}_i$. The independence of these measured accelerations from mass, which manifests itself in every case of a non-inertial observer (for example, an observer rotating about an axis), and the very fact of feeling the effects of the entrainment acceleration, allow one to recognize that the observer and the system to which they are bound are not inertial.

17 I have used boldface to indicate vector quantities.

18 It is the empirical fact from which Einstein developed General Relativity.

It is important to realize how the independence of relative accelerations from mass in a non-inertial system is a consequence of the very principle of inertia. Indeed, consider a system of bodies C initially all at rest relative to a reference frame S , but unconstrained to it and free from one another, so as to be able to move independently. Both the bodies C and the environment S initially move in uniform rectilinear motion with the same velocity $\mathbf{v}$ relative to an inertial or quasi-inertial system, such as the Earth's surface, O ; by the principle of inertia, all the C bodies, not being dragged by S , will persist in their uniform rectilinear motion relative to O with the same velocity $\mathbf{v}$ even if S undergoes an acceleration. The motion relative to the accelerated system is therefore a bulk motion, with the same acceleration relative to S . Inertia (the tendency of bodies to move with constant velocity) implies the independence of the acceleration relative to a non-inertial observer from the mass of the body. Conversely, if a set of free bodies manifests, relative to a given observer, a bulk accelerated motion independent of the mass of the individual bodies, we must deduce that such an observer is not inertial, and that the bodies in question are actually moving in uniform rectilinear motion relative to an I.O. By "bulk accelerated motion" one must understand not only the case where the accelerations of the individual bodies are equal, but also the case where they are subjected to centrifugal forces: in this situation, the accelerations will not be equal; being directed away from the axis of rotation and dependent on the distance from it, they will depend on the positions of the bodies. The accelerations measured by a non-inertial observer either depend on the positions of the accelerated bodies or are equal for all. It is appropriate to introduce the concept of an acceleration field, an example of which is the gravitational field, which however persists even if the observer is inertial¹⁹: therefore, while an acceleration field measured in a non-inertial system can be removed, this is not possible for the gravitational field.²⁰

In summary, all I.O.s are in uniform translational motion relative to one another, and all non-inertial observers are in accelerated motion relative to the I.O.s and possibly to each other. The latter are recognized by the fact that they measure acceleration fields due to their accelerated motion relative to the I.O.s, which are not observed by the latter.

19 In classical physics, a field is a quantity dependent on position. It is a piece of information contained in space. Applied to the acceleration of gravity, the concept suggests the idea of a relation with space, in addition to that of mutual attraction between bodies. There is more: the electromagnetic field propagates in a vacuum at the speed of light, and test electric charges undergo the action of the field, not that of the source charge; the latter might not even exist at the instant the test charges are acted upon by the field. In the same way, and for the same reason, we can receive light from a very distant star that has already exploded. The same holds for the gravitational field; the interaction between two masses is not instantaneous, and the gravitational field also propagates at the speed of light in a vacuum. A field of fictitious accelerations, on the other hand, carries no energy, exerts no forces, and has no physical reality; it is merely the product of observations conducted in non-inertial systems.

20 This observation also constitutes the starting point for the formulation of General Relativity.

5. Mach's Position

The physicist and philosopher of science Ernst Mach, after pointing out how Newton had not strictly adhered to the facts in formulating the concept of absolute space, proposed that all motion should be considered relative to a given body, or to a system of bodies, excluding from the analysis of phenomena any "metaphysical" hypothesis—that is, any hypothesis not comparable with observations: "*No one is competent to predicate things about absolute space and absolute motion; they are pure things of thought*" etc.; "*When we say that a body K alters its direction and velocity solely through the influence of another body K', we have asserted a conception that it is impossible to come at unless other bodies A, B, C. . . . are present with reference to which the motion of the body K has been estimated.*"²¹ But then, what would happen if the Newton's bucket were stationary and the distant stars were to rotate whirling around it? This question is a consequence of Mach's ideas on motion: all motions are reciprocal, and it is immaterial to ask which body is at rest and which is in motion. Thus, the centrifugal forces measured by the terrestrial observer would be the effects of some action exerted on the observer himself and on nearby bodies by distant bodies: this raises the problem of the nature of the forces produced by the remote masses of the Universe (that is, the fixed stars) on bodies rotating relative to them. This position may imply that inertial forces, or, if one prefers, fictitious accelerations, would be generated by the effect of the total mass of the universe on the individual masses that compose it; in fact, the effects of motion would be a sort of self-interaction of the entire cosmos.

It is difficult to avoid reaching this conclusion if one assumes that in physics the cause of every phenomenon must be understood as a force, that is, as the action of one body on another. What kind of forces would these be? One could say that these forces are the very origin of the inertia of bodies: indeed, it manifests itself as the tendency of a body to resist the forces that subject it to acceleration; but this is always relative to the remote masses, which represent the physical "background" of every event (and which replace Newton's space): therefore, inertia is the tendency to oppose a change in velocity relative to the remote masses (that is, to the ensemble of the fixed stars) and must be a consequence of the action of these masses (which are, in a sense, the Universe itself) on individual bodies. According to the most common formulation of the so-called Mach's Principle²², the inertia of a body is not an intrinsic property of matter immersed in an absolute space, but is determined by the distribution of all the mass in the universe. Hence, mass itself (which measures inertia) is no longer a property of bodies, but expresses the relation between bodies and the rest of the Universe.

In reality, the criticisms Mach leveled against Newton's interpretation of the experience of the water in the rotating bucket did not explicitly imply an explanation of inertial forces in the terms of dynamics, such as the one set forth above. Given his extreme empiricism, Mach did not propose any definitive explanation, limiting himself to setting forth what is possible to know experimentally. In his philosophical creed, science seems compelled to limit itself to organizing the information obtainable from observations and performable experiments. Even if this position has the merit of eliminating useless ideas, on the other hand it tends to reduce science to an essentially descriptive function, as if it were the map of a territory, renouncing—if carried to the extreme—what is most vital in scientific research, namely the proposal of hypotheses or general theories from which to deduce the explanation of individual phenomena within an organic framework. Without a theoretical apparatus there is no explanation, there is only description, and one does not see how a theory can consist solely of material of empirical origin.

21 From *The Science of Mechanics*, Open Court 1919.

22 In reality, Mach never formulated it as a "principle" nor did he ever give it this name. The name was coined by Albert Einstein in 1918, seeking to capture the essence of Mach's criticisms of Newton.

Let us now examine Mach's critique of Newton in more detail. *"All our principles of mechanics are [...] experimental knowledge concerning the relative positions and motions of bodies."* Further on, Mach considers the idea that all of space might be filled with a medium (*"the thing in itself is not impossible"*): should experimental knowledge on the subject increase, the idea of assuming an all-pervading medium as a reference frame could have greater scientific value than that of absolute space, *"which brings with it so many difficulties."*

"Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the Earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick." Therefore, *"...in the reflection here presented Newton has again acted contrary to his expressed intention only to investigate 'actual facts'"*. All this is perfectly adequate if we wish merely to describe what is observed, but it might not be sufficient if we ask ourselves what actually happens to justify the phenomena. Observation alone teaches nothing beyond the recording of what we have observed. Mach considers motion as such to be the immediate cause of centrifugal forces, but we cannot believe this to be the ultimate explanation. If motion is a relation between bodies, it is not difficult to take a small step further and ask whether masses—nearby ones and, above all, distant ones (the so-called fixed stars)—do not exert some influence on rotating bodies: otherwise, how could motion as such produce the observed effects? If all motions are relative, and if dynamics is based on the concept of force, the conclusion seems inevitable: centrifugal forces, and indeed all inertial forces, manifest the action of remote masses.

However, a little further on, Mach concedes that *"Newton, notwithstanding a certain propensity for the metaphysical absolute, always allowed himself to be guided by the caution of the scientist."* He agrees with Newton in recognizing the *"great value"* of the law of inertia, *"simple and precise,"* and observes that Newton's laws on the motions of the planets remain in accordance with the law of inertia even if these motions are referred to the heavens of the fixed stars. In this reference to material systems in the study of motions and the system of the world, Mach recognizes the *"caution"* of Newton the scientist.

Mach analyzes the principle of inertia on strictly empirical grounds. *"Bodies very remote from each other, which move with constant direction and velocity with respect to other distant fixed bodies, change their mutual distances proportionately to the time."* Indeed, every observer would agree with this description; however, Mach does not provide—in this context—any definition of *"fixed bodies,"* which, it seems to me, he considers to be fixed relative to the stars, which are themselves fixed by convention. In fact, the choice of the fixed stars as the ultimate material reference is consistent with Mach's empiricism, considering that nothing better is to be found. Yet, reflection on this very choice should reveal the limits of empiricism, given that the stars are not at rest at all; nevertheless, when one must judge the nature of a particular motion, remote masses seem to be the best reference system accessible to observation, even if one were to accept the idea of absolute space.

Mach proposes an original formulation of the uniform rectilinear motion of a mass n : its average acceleration relative to the remote masses m_i located at distances $\mathbf{r}_i$ from n is zero, that is:

$$\frac{d^2}{dt^2} \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i} = 0.$$

[This same equation also states that the center of gravity of the masses m_i in the reference frame of n is at rest relative to any body moving inertially, confirming the idea that inertial forces are due to acceleration relative to the center of gravity of distant masses or the fixed stars.]

In summary, Mach's approach consists in replacing a hypothesis (absolute space) with a point definable in mechanics, namely the center of gravity of distant masses or of the universe (nearby masses do not seem to play any role in the origin of fictitious forces). It is not possible, Mach argues, to abstract away from the rest of the universe. If we accept that gravity is the effect of the mutual action of two or more masses, we can accept, taking just a slight step beyond Mach's own thought, that fictitious forces are the effect of the action of all other masses upon a single one. This is essentially the meaning of the Principle attributed to Mach.

However, it seems clear to me that Mach was able to lucidly pose the problem of space and motion, but certainly not to solve it. The interactions between masses, as far as was known in his time, were described by Newton's Law, which appears entirely alien to Mach's Principle, at least at first glance; exploring more deeply, one can grasp an affinity between inertial forces and the force of gravity, since in both cases the accelerations, as we have seen, do not depend on the masses, but on position. The reader will note that this affinity recalls the principle of the identity of inertial and gravitational mass, well known since Newton, but which remained unexplained until Einstein formulated the Equivalence Principle. Mach's critique is essentially methodological in character: from his point of view, it was a matter of eliminating superfluous metaphysical hypotheses, limiting oneself to what can be empirically deduced.²³

The developments due to Einstein do not confirm Mach's Principle, even though Einstein drew inspiration precisely from his critique of Newton. What is preserved of Mach's approach in General Relativity is a formulation that does not require space as a background, in the sense that the substance of spacetime coincides with the totality of past, present, and future events. The theory establishes how the network of events is organized and links geometry to matter.

However, geometry is not completely determined by the presence of matter. Spacetime is endowed with characteristic properties such as curvature, and moreover, it is possible to conceive of a universe devoid of matter. Indeed, astronomical observations confirm that the distances between galaxy clusters are so vast that the observable universe can be considered empty on average. Masses move along the geodesics of spacetime, not through direct interaction with one another (there is no possibility of instantaneous interaction, a question Mach did not pose, because it is a consequence of the synthesis of mechanics and electromagnetism accomplished by Einstein himself with the formulation of Special Relativity); therefore, the motion of a mass in free fall depends on the local geometry of spacetime. Furthermore, the structure of spacetime is governed by transformation laws, which are defined by the mathematical properties of a four-dimensional Riemannian manifold and by the Principle of General Relativity, according to which the laws of physics are the same for all observers. In this respect, there is fundamentally no difference from the pre-relativistic concept of space, except that the geometry of a four-dimensional manifold replaces Euclidean geometry. From this point of view, General Relativity is rather an extension of Newton's theory, somewhat as Euclidean geometry is a limiting case of a non-Euclidean geometry, with the essential difference that it makes no sense to define any absolute motion; in this, Mach's position is fully embraced.

23 This position, of a substantially positivist nature, conceals a great difficulty inherent in the term "deduction." Evidently, it cannot be understood in the sense of "mathematical demonstration," which has nothing empirical about it; but on what basis can we affirm that all of physics is deducible solely from empirical observations?

There is a weak point, or at least an unclear one, in the thesis of the relativity of all motion as set forth by Mach. Since it is not possible to distinguish between the Earth's diurnal rotational motion relative to the fixed stars and the equally diurnal motion of the fixed stars about the extension of the Earth's axis, we should ask what dynamical consequences the second interpretation would have. The relativity of all motion makes every possible description equally plausible. Otherwise, one would have to accept either that some privileged body or system of bodies exists, as had always been thought, or that everything must be referred to space, or that every choice of a reference body is a falsification of reality—yet it is not clear how one could do without such a choice. In order to maintain that motion is only relative, since it is accepted that rotation relative to the fixed stars generates forces directed away from the axis of rotation, one must, by the very same reasoning, accept that the rotation of the fixed stars produces analogous effects. The reciprocity of all motion implies that the laws of dynamics are the same in all reference frames, without exception. On Mach's own premises, how can we rule out that the fixed stars farther from an axis of rotation would undergo centrifugal accelerations sufficient to drive them away from those closer to the axis? Nothing would have entitled Mach to assume that they constitute a rigid structure, like the celestial spheres of the ancient astronomers. Moreover, given that there are multiple rotating bodies in the Universe, how is it possible for the fixed stars to rotate about the rotational axes of each individual rotating body, depending on which one is chosen as the reference? It seems that the concept of motion relative to the fixed stars cannot be converted into the motion of the fixed stars relative to a body. Strictly speaking, it would seem that the relativity of motion, understood as indifference to any hierarchy among reference frames, when carried to its ultimate consequences, leads to the impossibility of regarding any physical system whatsoever as fixed. Consider the relative motion of the Earth around the Sun: the centrifugal force to which the Earth is subject—were one to consider only relative accelerations—would be far smaller than that experienced by the Sun, whereas the force of gravity acting on the two masses is the same. There is no difficulty if the accelerations in question are not the relative acceleration of one body with respect to the other, but are instead the absolute accelerations with respect to space or to some "fixed" system. But if all motions are relative, it seems impossible to identify a "fixed" system. The relativity of all motion is (almost) obvious in kinematics; but in dynamics?

Similar objections have been raised in some articles:

Hartman, H. I. & Nissim-Sabat, C., "On Mach's critique of Newton and Copernicus", arXiv:astro-ph/0311464. It is the article most closely aligned with the objections presented above.

Bhadra, A. & Das, S. C., "Comment on 'On Mach's critique of Newton and Copernicus'", the response to the previous one (available on arXiv, id 0705.1207).

Hartman, H. I. & Nissim-Sabat, C., "In Self-contradiction, Machian Geocentrism Entails Absolute Space", arXiv:1406.5434.

On the historical context (The Friedländer experiment): Lichtenegger, H. & Mashlohn, B., "Mach's Principle", arXiv:physics/0407078.

6. Time

Even more than space, time escapes precise definition in classical mechanics. Newton's ideas on the subject give precise form to ideas already present, albeit confusedly, in common sense²⁴ (*"I do not define time, space, place, and motion, for they are well known to all"*). We may ask to what extent Newton, in his conception of time, was influenced by pre-existing ideas not fully justified by experience — more so than in his conception of space, which was grounded in the mathematical knowledge of his era. In his time, there existed no theory of time comparable in scope and depth to geometry. Furthermore, time as understood in the West—linear, quantitative, homogeneous, measurable—is a specific cultural construction, to which multiple factors of different natures contributed: the Judeo-Christian tradition (linear time from Creation to Judgment), the Scientific Revolution (time as a measurable quantity), and the construction of mechanical clocks. Conversely, we owe to the consequences of subsequent technical developments the establishment in industrialized countries of a quantitative, economic conception, tied to the use of resources, the rationalization of economic activities, and so on.

Ordinary language essentially manifests two properties attributed to time: irreversibility ("we cannot go back in time") and ordering; there is a before and an after. We can add one-dimensionality, already implicit in characterizing the succession of instants or time intervals with only the categories of "before" and "after". The differences with space are many, and they are not well grasped in classical dynamics. In reality, until the formulation of Special Relativity and, more precisely, Einstein's critique of the concept of simultaneity, time was not seriously analyzed by physicists.

Comparison with space provides some information of a predominantly negative character: we can study parts of space with the tools of geometry and discover many properties, but we cannot perform analogous operations on time. In some way, it is possible to treat space as a whole, albeit abstractly, but one cannot proceed similarly with time. Time is not operationally divisible, in the sense that it is not possible to separate a part of it as an interval between two instants, preserve it, and study it carefully. This kind of operation is not performed on time itself, but on a suitable representation of time — one that permits it and is geometric in nature. It is possible to traverse a closed path, but there is no return to the starting instant. Newtonian dynamics is not capable of treating irreversibility; one must resort to the second law of thermodynamics, because classical mechanics is reversible. We can start from this aspect to explore some difficulties encountered when trying to analyze time within a purely classical framework.

Newton's second law is invariant under time reversal. This term means nothing more than the substitution of $-t$ for t in the equations, starting from the second law. This is understood to mean that the laws of classical physics do not depend on the direction of time's flow, or rather, since time does not flow at all²⁵, on the direc-

24 It seems that some aspects of time are universally shared by all human cultures: all distinguish between "before" and "after"; all human beings perceive that some events last longer than others; all human cultures (and most living organisms) are synchronized with the day/night cycle; all cultures have memory of past events. This is a minimal set of conceptions; on many other aspects significant differences emerge. It is evident that the "pure" physical aspect of time is always mixed with something psychological or culturally characterized; it would seem to reduce to a succession of configurations of objects in space and physical quantities that never repeat. Here I limit myself to the representation offered by classical, non-relativistic mechanics.

25 This representation of time as a continuous flow is as common as it is illusory and, to some extent, has penetrated classical mechanics as well. It is in truth a very strange image, because it attributes to time an action such as "flowing" which in physics should be attributed only to material entities. Time is not an observable entity, it is not a phenomenon, it is not anything that can be grasped in its entirety. To say that time flows means to observe that things and ourselves change. We can add "in time", almost as if we were contained in something we call time. This representation, in particular, fits well with space; it is more difficult to establish whether it is adequate to our relationship

tion in which we order the phenomena that occur in it, while remaining of the opinion that time contains all events as space contains all bodies. Furthermore, if it is possible to predict the future behavior of a system of bodies from certain sufficient information, it is therefore possible to establish what its past configuration was, instant by instant. This is a critical point: the universe is irreversible, since apparently it does not return to a previous configuration, but classical dynamics does not forbid anything of the sort.

The representation of time in classical dynamics is a "parametrized" representation, in the sense that time is treated as a continuous succession of instants, each of which without extension, as if it were a point of time, and therefore associable with a number by counting time starting from a certain instant. Imagining time as an ordered succession of instants is a very strong hypothesis, if only because it implies that it is indefinitely divisible, or rather, constituted by a succession of time intervals that are pairwise disjoint and can be arbitrarily small. In brief, the time interval is treated as an Archimedean quantity, exactly like lengths and so forth. However, in practice this procedure works well and has prevailed against every philosophically sensible objection. In fact, the problem at the root of the method is not the "spatialization" of time, an operation that is not really performed insofar as it is not performable and has only a geometric meaning. It is not possible to transform time into space, but only to represent mathematically time and its parts in the same way as half-lines, segments, and so on. The real problem is the assumption, arbitrary and experimentally neither verifiable nor falsifiable, that it is composed of a succession of indefinitely reducible intervals. This holds for space as well. However, for all that it is an entirely arbitrary hypothesis, it does not in fact disturb the edifice of classical dynamics, because there exist experimentally detectable phenomena that occur in times much, much shorter than those occupied by the phenomena treated in classical physics.

Treating the instant—an entity that is not really such—as the fundamental parameter in classical dynamics is a necessity, since the best method to describe the motion of a body is to assign to each instant a position defined by equations of motion of the type $x_i = f_i(t)$. This allows motion to be studied analytically, and it is clear that the possibilities offered by analysis have prevailed over any conceptual reservation on the subject, however well-founded it might have been.

Strangely, many reflections on absolute or relative time have not grasped the fundamental point, namely that until Einstein it was thought that all observers, regardless of their physical state, should agree on the fact that a certain event occurs at an instant that is the same for everyone, and that the time interval separating two instants is the same for everyone. It is difficult to establish whether one should be more astonished by Einstein's discovery of the relativity of time or by the fact that no one thought of it before.

The rooting of the idea that time flows the same way for everyone is undoubtedly linked to the appearances suggested by ordinary experience. The daily and seasonal rhythms, the recurrence of the same phenomena at regular time intervals or perceived as such, the very marking of daily life impose themselves with such force as to exclude anyone seriously posing this problem. Just as everyone is convinced that a meter is a meter for everyone, so it holds for the second and for any time interval. But there is a hidden aspect, linked

with time. When it comes to this, it is inevitable to allude to the representation that everyone may have of it, which partakes of common sensations and ideas suggested by personal reflection. The fact is that space has a defined structure, that of geometry. In a sense, it is objectively definable, even if it is not certain that the perception of space is common to all cultures and individuals, but it is much more difficult to establish what the elements are that objectively define time. The naive idea of the flow of time is an effective image of a current that drags everything and everyone along, and that never reverses the direction of its movement; it is a simple way of saying that the world of experience is irreversible and is intimately linked to the very destiny of living beings. Classical physics does not define time; it simply tries to represent the evolution of physical systems as a function of the instant, the equivalent of a point in the dimension of time.

precisely to the unit of measurement. How can we establish that a given time interval like a second, of which we have a precise definition, between a certain instant and a subsequent one, is equal to that second separating two other instants? One possible answer is that every time interval has a constant ratio with others, but this implies that time measurements are ultimately reducible to the invariance of the ratios between the times corresponding to cyclic events; in fact, all time measurements go back to the observation of phenomena that apparently repeat in a regular manner. If we accept this analysis, we must agree that the possibility of measuring time is founded on the cyclicity of certain phenomena that manifest themselves on very different scales (celestial motions, oscillations, and so forth) and on the empirical observation that the ratios between the times in which they are accomplished seem to be constant.

The measurement of lengths and so forth seems to pose fewer problems; however, there is an analogy with the measurement of time intervals. Indeed, we can doubt that a given length, such as that of a rod, is always the same for everyone, or does not change with time, just as one can doubt that every time interval chosen as a unit of measurement always has the same extension, and the reason why nobody or almost nobody poses these problems is exactly the same one that has already emerged for the measurement of time: empirically, nothing suggests the contrary. Or, reasoning from an epistemological point of view, we could admit with Kant that space and time are *a priori* forms of sensibility; there is not, I believe, any way to establish which approach is the true one, since every experimental verification presupposes—if Kant is right—those very preconditions of sensibility that one should investigate experimentally. From the physical point of view, one should take into account only experience, or rather the information provided to us by experimental investigations. However, these are not immediate observations by purely receptive subjects. The information is the result of complex procedures and of many factors, and no experiment can establish whether the unit of measurement of time or of length is a constant. It is not even clear whether this problem makes sense, precisely for these same reasons.

It is therefore evident that the common ideas about space and time, and also the less common ones rooted in Newtonian dynamics, are prejudices whose solidity is founded on ordinary experience and on generally accepted conventions, and not on a thorough examination. The gaze of those who deal with physical sciences is turned toward the discovery of novelties rather than toward reflection on foundations, to which one turns when previous ideas manifest evident insufficiencies.

7. Special Relativity and the Equivalence Principle

An obstacle to the renunciation of an absolute reference frame endowed with physical reality came from electromagnetism. Maxwell's equations are verified by observers for whom the speed of light c is that calculated from the absolute dielectric constant of the vacuum and the magnetic permeability of the vacuum. In practice, given the high value of c , the possible motion of the experimenter or the source does not influence the outcome of most experiments, except for the variation in the oscillation frequency of the electromagnetic field when the observer and the source are in motion relative to each other (the Doppler effect, already demonstrated for sound waves). However, if the question is considered from a theoretical point of view, one finds that the equations of electromagnetism are not invariant under Galilean transformations, which describe the transition from one reference frame to another in uniform translational motion relative to the first. Interpreted within classical kinematics, Maxwell's theory thus seemed to require a privileged reference frame: the one at rest relative to the luminiferous ether, the medium in which electromagnetic waves were supposed to propagate. In systems moving relative to the ether, the speed of light would have had to be different from c , and the equations would not have preserved the form given to them by Maxwell. Maxwell's theory, not compatible with the Galilean-Newtonian transformations, would then be rigorously applicable only to the class of observers at rest relative to the medium of light propagation, since not all inertial observers satisfy this condition. Indeed, at the time (the second half of the nineteenth century), it was not conceived that a wave could propagate through empty space; the properties of the medium that propagated light were seriously discussed and found to be quite strange and singular: the ether—as the medium was called—was supposed not to interfere at all with the motion of bodies, while being extremely rigid, and so forth.²⁶

Maxwell's equations, in the conception of their author and his contemporaries, were valid in the reference frame in which the luminiferous ether was at rest. Maxwell himself, in deriving his equations, implicitly assumed that the source and the observer were at rest relative to the ether, and he knew that relative motion with respect to it would have to produce observable effects on the speed of light.²⁷ Only some time after the Michelson-Morley experiment²⁸ (1887) was it recognized that there was no need for any "ether" to propagate light. Today we know that Maxwell's equations are valid in all inertial frames, but this understanding was achieved only thanks to Einstein's Special Relativity. The definitive overcoming of the old concept of a privileged reference frame occurred when Einstein proved the invariance of Maxwell's equations under the Lorentz transformations: in fact, by formulating Special Relativity to conserve the equations of electromagnetism in every inertial frame, he renounced absolute time (the measurement of time intervals depends on the observer). It is immaterial whether the observer is in motion relative to the source, or vice versa; the observation of luminous and, more generally, electromagnetic phenomena depends only on their relative motion. The propagation of electromagnetic radiation occurs under different conditions than that of sound waves; the latter must propagate in a medium (air, water, etc.) and one must distinguish the three cases in which only the listener is in motion relative to the medium, or only the source, or both. Instead, the relative motion of the source toward the observer modifies the oscillation frequency and the wavelength of the radiation, without it being possible to distinguish between the two motions relative to any medium of propagation.

26 The term, if not the concept, has survived to the present day in expressions such as "communications via the ether", etc.

27 See D. Kaiser, "Lecture Notes for 8.225/STS.042, 'Physics in the 20th Century': Electrodynamics for Maxwell and Lorentz", published online at [Lecture Notes: Electrodynamics for Maxwell and Lorentz](http://www.superzeko.net).

28 The experiment aimed to measure the velocity of the Earth relative to the ether. No evidence of a motion of the Earth relative to a luminiferous medium filling space was found.

Furthermore, attention must be paid to the "pseudogravitational" character of fictitious forces, also called inertial forces, measured in non-inertial systems. According to the Equivalence Principle (Einstein 1907), a reference frame in free fall in a gravitational field is physically equivalent to an inertial reference frame in the absence of gravity. Put differently, all bodies contained within a freely falling environment fall with it with the same acceleration and the same velocity at every instant (if initially at rest relative to the environment); therefore, relative to an observer at rest with respect to the environment, they are in a state of rest. Hence all bodies in free fall in the same region of space fall with the same acceleration, which is a reformulation of the experimental principle of the identity of inertial and gravitational mass. To be precise, as set forth above, the principle is strictly true for a system in free fall within a uniform field, so as to exclude the appearance of the effects of tidal forces, that is, the forces acting on bodies immersed in a non-uniform field due to the differences in the intensity of the force of gravity; or, more realistically, for a system of dimensions small enough to render the spatial variations of the gravitational field negligible.

We have alluded to the relation of mutual implication that links the Equivalence Principle to the identity of inertial and gravitational mass. Let us start from the latter. In general, the second law states that $\mathbf{F} = m\mathbf{a}$,

where m is the inertial mass; indeed $a = \frac{F}{m}$ implies that acceleration and mass are inversely proportional.

We can apply it to the force of weight, $\mathbf{P} = m\mathbf{g}$, in which however m is not inversely proportional to g ; increasing the mass, the weight increases and the acceleration does not vary. The mass that appears in the law of universal gravitation is gravitational, since it is the source of the force of gravity. Inertia and gravitation are quite distinct phenomena, therefore conceptually the two aspects of mass must be considered as different quantities. Distinguishing the two masses, and applying Newton's second law to the force of weight, we obtain $m_i a = m_g g$ with $a = g$ (in absolute value), that is, $m_i = m_g$. Or, consider a force F , which impresses an acceleration a , balancing a weight P . The resulting acceleration is zero, therefore $a = g$; again, we find $m_i a =$

$m_g g$, and so on. We could avoid the conclusion $m_i = m_g$ by imposing $F = \frac{m_i}{m_g} P$, against the idea that equal

forces applied to the same body impress equal accelerations; but then, on the basis of what criterion could one establish the equality between forces in dynamics? Certainly not by applying the second law. The calculation of the effective force of a body sliding without friction on an inclined plane, the independence of the period of oscillation of a pendulum from its mass, the law of gravitation itself, and so forth, imply the identity of the two masses, not merely direct proportionality.²⁹

29 The identity between inertial mass and gravitational mass is not a convention, but a necessity for the coherence of the second law: when forces of different nature are composed (for example, an applied force and weight), the resultant is a single vector, and the acceleration is obtained by dividing by a single mass. If the two masses were not identical, this operation would not be possible in a coherent way. On the other hand, if the reader doubted that the relation between the two masses is one of simple proportionality, and that their identity is only a convention (for example, because the same unit of measurement has been chosen to be employed for both), let him consider that no one has ever proposed or performed an experiment to determine the constant of proportionality, and let him evaluate how many physical laws would change if the two masses were not identical.

If two quantities of different physical meaning coincide universally, we have, if not an outright contradiction, a conceptual inconsistency, since a mere lucky coincidence must be ruled out. There must be a deep connection, not evident in classical dynamics, between the two aspects of mass, and the manifestation of this identity is strictly linked to the Equivalence Principle. Let us suppose that, relative to an observer located inside a system and having no access to information from the outside, all the masses contained in the system and free to move accelerate with accelerations that are equal in absolute value, direction, and sense. The observer can interpret the motion of the bodies in two ways: as an effect of the acceleration of the reference frame (inertial forces) or as an effect of a uniform gravitational field. Due to the identity of inertial and gravitational mass, the two interpretations are physically indistinguishable.

In this example we have considered translational motions, but we can apply the principle to rotational motions as well. Consider the technique employed to simulate gravity inside a spacecraft placed at a sufficient distance from any mass, so as to be in a gravity-free environment. By setting the environment in rotation about an axis of symmetry, centrifugal forces will push objects and observers inside it toward the walls. Let d denote the distance from the axis of rotation and ω the angular velocity. Equating the centrifugal acceleration $a = \omega^2 d$ and the acceleration of gravity g , we obtain $\omega = \sqrt{\frac{g}{d}}$, an expression analogous to the formula for the angular frequency of a simple pendulum.

Let us return to translational accelerations. Suppose the observer is in the spacecraft, and that an acceleration of magnitude equal to g relative to an inertial frame is now imparted to it. The observer is pushed toward a wall that acts as a supporting surface; suppose a mass is connected by an inextensible string to the opposite wall, and that initially the string and the direction of motion, aligned with the acceleration, form a small angle. The acceleration of the mass measured by the observer in the spacecraft, oriented in the same direction but opposite in sense to the motion of the environment, is completely indistinguishable from that of gravity, and the suspended mass will behave exactly like a pendulum on Earth.

In summary: the behavior of fictitious forces, the Equivalence Principle, and the identity of inertial and gravitational mass³⁰ are different aspects of the same fundamental law. The independence of fictitious accelerations from mass implies that a system in accelerated translational motion relative to an inertial frame generates within it a field of apparent forces that is locally indistinguishable from a uniform gravitational field.³¹

What are the consequences of the Equivalence Principle? We can explore them by considering the observations performable in a system in free fall. In fact, if we neglect the effects of the Earth's diurnal rotation, all

30 It is remarkable that Newton masked this identity by introducing the concept of mass. Indeed, he considered mass as the measure of the "quantity of matter," and on the basis of this concept he justified both the Second Law of dynamics and the law of gravitation, without being obliged to otherwise explain the deep connection between inertia and gravitation. Yet he was perfectly aware of the problem; in the *Principia*, Newton experimentally measured that gravitational mass (that which determines how much a body is attracted by gravity) and inertial mass (that which determines how much a body resists acceleration) are equal to a precision of about 1 part in 1,000.

31 Three slightly different formulations of the Equivalence Principle have been proposed:

1. Weak Equivalence Principle (WEP): All bodies in free fall undergo the same acceleration, regardless of their composition or mass. (This is what Galileo, Newton, and Eötvös verified);
2. Einstein Equivalence Principle (EEP): In a sufficiently small region of spacetime, the laws of physics in a freely falling frame are those of Special Relativity. (This applies not just to mechanics, but also to electromagnetism, thermodynamics, etc.);
3. Strong Equivalence Principle (SEP): Like Einstein's formulation, but it also includes gravitational energy itself: a body whose mass is partly due to its own gravitational binding energy falls with the same acceleration as any other body.

experiments performed on the surface of the planet are of this kind. Were it not for the diurnal rotation, it would not be possible to distinguish the terrestrial observer from an inertial observer. No experiment performable in a laboratory can reveal the gravitational attraction of the Sun, since all bodies present undergo it to the same degree. The Earth and the Moon both move around the Sun, but viewed from the Earth the Moon's orbit is nearly circular, as if the solar attraction did not exist. The only effect that an inertial observer would attribute to a body rotating around a central mass would be due to tidal forces; but the freely falling observer could simply note that the central mass deforms the system in which he finds himself and generates local relative accelerations, just like the tides generated by the Moon. In this respect, there would not be a great difference between the heliocentric and the geocentric system.

However, there is a most important consequence, which leads us to scenarios very far removed from pre-relativistic classical physics. The gravitational field, unlike the fields of fictitious accelerations, is generated by masses and not by the motion of the observer, that is, of the system to which he is connected. By passing to an inertial system, these accelerations vanish. Their appearance or not depends on the coordinate system; a reference frame rotating relative to another defines a different metric (that is, the generalized distance that separates two events in spacetime). Performing the calculation starting from an inertial reference frame in Special Relativity, one finds that the metric relative to the system in uniform rotation is not that of a "flat" spacetime. The curvature of a rotating system is apparent: it can be eliminated by passing to an inertial system. In the case of a gravitational field, on the other hand, the curvature is real and cannot be eliminated by any coordinate transformation, except locally and approximately.

The passage from the classical to the relativistic description implies a profound revision of the concepts of space and time. They can no longer be considered independent of one another, and the integration of gravitation into the structure of space implies the abandonment of Euclidean geometry. It remains to be seen whether this revolution has entailed the definitive negation of the idea of a space independent of matter, such as Newton's absolute space, or whether it has replaced the latter with a purely relational structure, a set of rules for composing the totality of events into a coherent scheme.

8. Conclusions

General Relativity—the theory generally accepted today as the best description of spacetime and gravitation—does not require a background; in short, it is a general theory of reference frame transformations applied to a continuous and differentiable four-dimensional manifold, in which the coordinates, representing point-events, have no meaning except within a given reference frame. There is no background independent of events, and it is possible to determine the position of a body only relative to a reference frame that can be chosen in an absolutely arbitrary manner. Physical laws must be valid in every possible reference frame; that is, the equations expressing them must transform in such a way as to maintain the same form. Thus, these equations and their related transformation laws constitute the architecture that unifies space, time, matter, and energy.

However, space—or rather, spacetime—possesses local and global properties that, in my view, do not allow it to be treated as a purely relational system. Scalar curvature, for example, the structure of geodesics, and so forth, are actual physical properties; globally, the expansion of the universe revealed by the recession of galaxies is understood as an expansion of space itself; finally, space itself contains energy by virtue of the cosmological constant. The recent discovery of the increase over time in the expansion rate of the universe does not concern merely a set of relations; it is a global property of space, like the characteristics listed above. No relational theory justifies the existence of space, even if it were to manifest no physical characteristics other than extension and geometric properties.

Further evidence in this direction can be deduced by examining Einstein's equations. It is generally stated that these establish how matter determines geometry, and how the latter guides the behavior of matter. This description, highly effective for its conciseness, must be supplemented: among the solutions to the equations—that is, among the universes compatible with the theory—we find models devoid of matter, and indeed our own universe is almost completely empty. A list of positions, velocities, and relative accelerations among the various parts of the cosmos is not the cosmos.

Quantum theory, on the other hand, requires spacetime as a background. Being applied to microsystems, it does not by itself imply any theory of space; it simply presupposes it. In short: although many have striven to reduce space to a set of relations and to demonstrate the uselessness and inconsistency of any absolute reference, the very ultimate outcomes of their critiques and of the knowledge accumulated especially in the twentieth century imply that space maintains a physical consistency of its own, independent of the presence or absence of matter. If there is any doubt about this, let us consider Einstein's equations; they have solutions even when the energy-momentum tensor is imposed to be zero throughout spacetime.

The necessary condition for obtaining a definitive solution to the nature of space and time is the elaboration of a theory that unifies General Relativity and Quantum Mechanics. We have candidates such as *String Theory* and *Loop Quantum Gravity*, but the fundamental problem is not only their experimental verification. It is not possible to know whether "our" universe is the only one, and it is extremely doubtful that any proof can be produced in this regard. In particular, it is not possible to know whether the characteristics that determined the history of the universe are arbitrary initial conditions, or whether they are selection parameters that identify one universe among many existing ones.

Furthermore, it is not possible to establish whether every spacetime has the same structure as the one in which we are immersed, or whether there exist n -dimensional manifolds that are physically consistent but of a completely different nature. We cannot know whether every mathematically consistent model (and there are infinitely many) describes a "real" universe, or what the number of these models is, and at this point it is

not even clear whether terms belonging to the philosophical rather than the physical lexicon, such as "to exist," "reality," and so forth, have a definite meaning when it comes to mathematical possibilities that go far beyond experimentation. We are, however, certain that Einstein's theory is far superior to the pre-relativistic one, which is a limiting case of the former.

Finally, General Relativity does not imply that all observers are equivalent, or that the description of relative motions is merely a matter of convention and convenience. The theory only asserts that it is possible to express all fundamental physical laws in covariant form, since they are independent of the choice of coordinate system; which implies that place cannot be defined except through an arbitrary choice of coordinates: for these have no physical consistency. We can refer to the Sun or to the Earth, but the description of the universe does not depend on the choice made. Freely falling systems are not equivalent to those subject to non-gravitational forces; they simply replace inertial observers. In a sense, the gravitational field, or the metric field, takes the place of the set of inertial systems, which in turn replaced absolute space. In cosmology, the arbitrariness of the choice of reference frame is replaced by a particular choice of the metric that takes into account the isotropy and homogeneity of space. The cosmological system describes galaxies as points on an expanding three-dimensional hypersurface, characterized by local motions; the universe has a certain age, which is the same for all galactic observers and—at least in the simplest version, which excludes local differences in the expansion rate—evolves homogeneously.³² The cosmic background radiation is another privileged system, despite not being a material body. By measuring its frequency, we can establish the instantaneous velocity of the Earth or of any object in space relative to the background radiation. In some way, the idea of an absolute reference survives, albeit in a form completely different from that of the ancient astronomical systems or from that given by Newton.

32 In standard relativistic cosmology (based on the FLRW metric), it is assumed that the universe, on sufficiently large scales, is homogeneous and isotropic. This allows four-dimensional spacetime to be ordered according to a succession of three-dimensional spatial sections, each labeled by a value of cosmic time (the proper time measured by a "comoving" observer, that is, one that follows the average flow of the galaxies). The galaxies, in this description, are approximately fixed (comoving) points on these three-dimensional sections, and what changes over time is the scale factor that determines the distances between these points.