THE EFFICIENCY OF HEAT ENGINES:
An Introduction to the Second Law of Thermodynamics

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1. Introduction

A **heat engine** can be defined as any device that converts heat into mechanical work through a continuous process (or, more precisely, a series of thermodynamic cycles) over an indefinite period. This implies that the engine must be capable of producing a potentially unlimited amount of work over an extended time.

From a purely physical standpoint, the specific mechanical structure and technical details of a given heat engine are of secondary importance. What matters is that the work is performed by a **working fluid** that expands and does work upon absorbing heat. This process can be achieved in various ways, such as by moving a piston or driving a turbine. Thermodynamically, a heat engine is defined by the physical processes occurring within its working fluid.

Clearly, the mechanical processes within a heat engine must be cyclic. This is necessary both because engines typically drive rotary mechanisms (such as crankshafts or gears) and because a fluid cannot produce work by expanding indefinitely; since the working fluid produces work by expanding, it must eventually be returned to its initial volume. Therefore, a heat engine must operate through a series of thermodynamic cycles to deliver an arbitrarily large amount of work over an extended time.

According to the First Law of Thermodynamics (Conservation of Energy), this work must be generated from heat. Consequently, there must be at least one **heat reservoir** (or heat source) to supply the required thermal energy. Since the engine's potential work output is unlimited, this reservoir must be able to supply arbitrarily large amounts of heat without its temperature dropping. Ideally, this means the heat reservoir must have an infinite heat capacity, allowing it to continuously supply heat without any change in temperature. In thermodynamics, such an ideal heat source is referred to as a **thermal reservoir**.

In practice, real engines - such as internal combustion engines in vehicles - are powered by combustion processes that irreversibly alter the chemical composition of the working fluid, which must be replaced at the end of each cycle. Thus, the actual "heat source" is a continuously repeated chemical reaction rather than a persistent physical system.

From an ideal standpoint, the ultimate heat engine would produce work by converting heat extracted entirely from the surrounding environment - exploiting the enormous thermal energy contained in the atmosphere, the oceans, and so on. Such a machine would convert the absorbed thermal energy entirely into work while in thermal equilibrium with the environment, requiring no other heat sources (i.e., exchanging heat with a single reservoir).

2. Efficiency and the Kelvin-Planck Statement

In practice, during a single operating cycle, any heat engine absorbs a certain amount of heat Q_1 from a "hot" reservoir (i.e., one at a temperature higher than the ambient environment) and converts only a fraction of this thermal energy into work W. The remainder is dissipated - directly or indirectly - as unusable and unrecoverable waste heat, which is rejected to the external environment, acting as the "cold" reservoir.

Let Q_2 denote the amount of heat rejected to the cold reservoir. According to the First Law of Thermodynamics (the principle of conservation of energy), assuming both heat and work are expressed in the same units, the net work produced during a single operating cycle is given by the equation:

$$\boxed{W = Q_1 - Q_2}$$

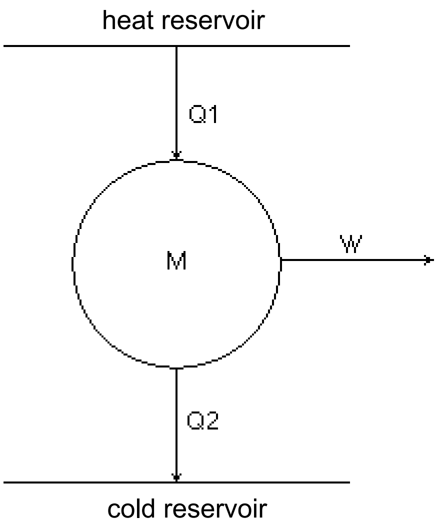
The rejected heat Q_2 can never be zero; therefore, the ratio of work output to heat input is always strictly less than 1. By definition, this dimensionless ratio is the thermal efficiency (denoted by the symbol η) of the heat engine. It is often expressed as a percentage, where 1 corresponds to 100%. Thus, by definition:

$$\boxed{\eta = \frac{W}{Q_1}}$$

The Second Law of Thermodynamics stems from the realization that this fundamental limit on the efficiency of any heat engine is not merely the result of unavoidable technical difficulties or the practical impossibility of perfectly insulating an engine. While it is true that real engines overheat if they do not reject heat to the environment (making practical thermal losses unavoidable), the absolute impossibility of achieving 100% efficiency is a matter of fundamental physical principle, entirely independent of engineering constraints.

To formalize this, we introduce the **Kelvin-Planck statement** (often referred to simply as Kelvin's postulate), which states: *It is impossible for any device that operates on a thermodynamic cycle to receive heat from a single thermal reservoir and produce a net amount of work.* In other words, it is fundamentally impossible to build a cyclically operating heat engine that converts all the heat extracted from a uniform-temperature thermal reservoir entirely into work.

The schematic energy-flow diagram of a heat engine is as follows:



3. Irreversibility and the Clausius Statement

To verify that this is a matter of fundamental principle - and not merely a technical difficulty - we prove that the Kelvin-Planck statement (KP) is logically equivalent to an even more self-evident physical principle. In particular, we can verify its connection with the irreversibility of heat exchange.

A phenomenon is said to be **irreversible** when it evolves in a definite direction in time and cannot be reversed. More precisely, in an irreversible process a physical system evolves from an initial state to a final state through an ordered, continuous sequence of instantaneous configurations, yet it is impossible to swap the initial and final states simply by reversing the order of all the phases of the process. Many natural processes are irreversible - in fact, all physical processes are, with reversibility being merely a limiting case - and heat exchange is among them. Indeed, when two bodies at different temperatures are placed in thermal contact, heat flows "spontaneously" from the warmer body to the colder one, whereas the reverse cannot occur spontaneously. In the absence of external intervention, the thermal contact between the bodies tends to reduce the temperature difference until a state of uniform-temperature equilibrium is reached.

That said, it is not absolutely impossible to transfer heat from a cold body to a hot body while maintaining or even increasing the temperature difference between them: this is precisely the process carried out by **refrigerators**, which therefore generate a heat flow opposite to the natural one. This, however, is an artificial process, which can occur only thanks to the work supplied to the refrigerator by an engine (generally an electric motor), forcing heat to flow from the cold reservoir to the hot one (the external environment).

From a thermodynamic standpoint, the energy flows in a refrigerator are the reverse of those in a heat engine: the latter converts part of the heat absorbed from a hot reservoir into work and rejects the remainder to the cold reservoir (the external environment), whereas the former absorbs heat from a cold reservoir and work from an engine, and rejects their sum to the hot reservoir (the external environment).

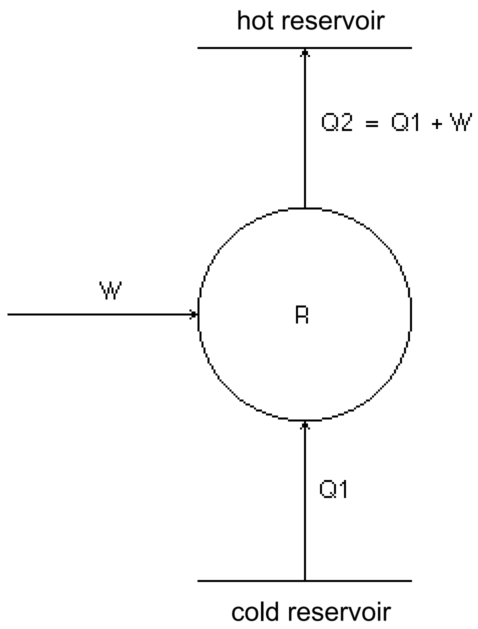
The energy balance has the same form as equation (1),

$$W = Q_1 - Q_2$$

where Q_1 is the heat absorbed from one reservoir (hot or cold) and Q_2 is the heat delivered to the other reservoir. All quantities are taken as positive.

The principle according to which heat flows "spontaneously" from hot bodies to cold bodies is known as the **Clausius statement** (CP): *a transformation whose only result is the transfer of heat from a cold reservoir to a hot reservoir is impossible*. Therefore, it is impossible in principle to build a refrigerator operating on a thermodynamic cycle without supplying external work. This principle would seem completely self-evident were it not for the phrase "without external work", which is precisely what clarifies the physical meaning of "spontaneously".

The schematic energy-flow diagram of a refrigerator is as follows:



4. The Equivalence of the Kelvin-Planck and Clausius Statements

There is a mutual implication between the Kelvin-Planck statement (KP) and the Clausius statement (CP), in the sense that if one of them were false, the other would have to be false as well. For instance, we can show that denying KP implies denying CP; that is, if it were possible to build a heat engine with an efficiency equal to 1 (KP false), then it would also be possible to transfer heat from a cold reservoir to a hot reservoir without any external work (CP false). Therefore, although they appear to be quite independent of each other, the two statements are physically equivalent, and both can be regarded as particular formulations of the Second Law of Thermodynamics (there are many ways to express the Second Law, depending on the particular aspect under consideration).

Step 1: KP false ⇒ CP false

Let us deny KP; this implies that a heat engine "M" is able to convert into work the entire amount of heat Q_1 extracted from a single heat reservoir at temperature T_1. We therefore obtain the equation:

$$W = Q_1$$

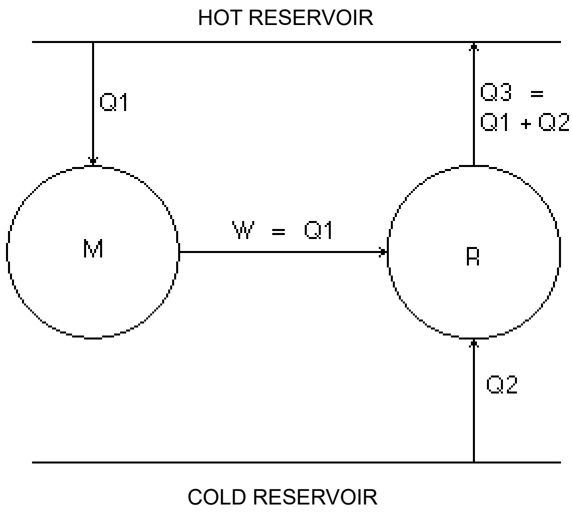
We can use this work to drive a "normal" refrigerator, which extracts heat Q_2 from a cold reservoir at temperature T_2 and rejects heat to the same hot reservoir. Denoting by Q_3 the amount of heat rejected by the refrigerator to the hot reservoir, conservation of energy gives:

$$Q_3 = Q_2 + W$$

which implies:

$$Q_3 = Q_1 + Q_2$$

$$Q_3 - Q_1 = Q_2$$



The last equation states that the net heat exchanged between the hot reservoir and the composite system consisting of the heat engine and the refrigerator, namely $Q_3 - Q_1$, is exactly equal to the heat Q_2 removed from the cold reservoir. Now let us consider the combined device as a whole, operating between the two reservoirs: the sole result of this hypothetical process would be a transfer of heat Q_2 from the cold reservoir to the hot reservoir without any external work (the work is produced and consumed entirely within the system).

Therefore, by denying KP we obtain a refrigerator operating in violation of CP; that is,

KP false ⇒ CP false.

Since we accept the Clausius statement as true, it follows by contradiction that the Kelvin-Planck statement must also be true.

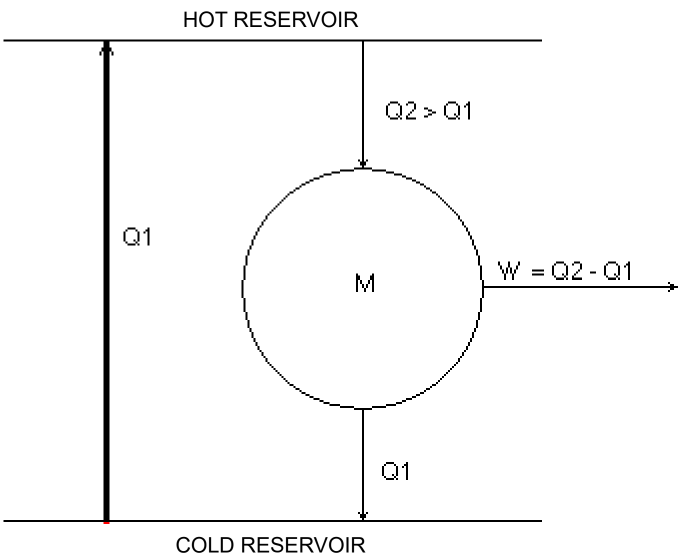
Step 2: CP false ⇒ KP false

The logical order of this proof can be reversed by showing that denying CP implies denying KP. That is, if a "spontaneous" transfer of heat from a cold reservoir to a hot reservoir were possible, then it would also be possible to operate a heat engine that converts into work the entire heat absorbed from the hot reservoir. In short, we can show that:

CP false ⇒ KP false

This means that the two statements are logically equivalent, in the sense that they must both be true or both be false - one cannot be true while the other is false. Hence, both can be regarded as equally valid formulations of the same principle.

Indeed, let us suppose, for the sake of contradiction (denying the Clausius statement), that an amount Q_1 of heat flows "spontaneously" from a cold reservoir to a hot reservoir. Now imagine that, during a single operating cycle, a "normal" heat engine extracts from the same hot reservoir a larger amount Q_2 of heat, converts the difference $Q_2 - Q_1$ into work, and rejects Q_1 to the cold reservoir, as an ordinary heat engine does. The net heat exchanged with the cold reservoir is zero; therefore, the overall process has converted into work the entire heat extracted from a single heat reservoir, in violation of the Kelvin-Planck statement. Hence, denying the Clausius statement implies denying the Kelvin-Planck statement.



An even more paradoxical variant is to suppose that the engine absorbs Q_1 from the cold reservoir and rejects Q_2 to the hot one, with $W = Q_1 - Q_2$. This scenario is particularly striking, since the heat $Q_1 - Q_2$ converted into work would effectively be extracted from the cold reservoir.

5. Real and Ideal Heat Engines

In theory, a heat engine can be either reversible or irreversible. An ideal engine cycle operating between two reservoirs would be reversible, since irreversibility implies thermal losses and the dissipation of work into heat rejected to the environment (the "cold" reservoir). Hence, a real machine is less efficient than an ideal one, in accordance with the inequalities

$$\eta_{IRR} < \eta_{REV} < 1$$

which apply to engines operating between the same reservoirs.

Concluding Remarks: Irreversibility and the Second Law of Thermodynamics

The considerations made so far about the nature of heat reservoirs and the operation of heat engines are actually very schematic. To begin with, a heat source is not necessarily a true thermal reservoir, nor is it physically external to the machine; for example, in common internal combustion engines (such as automotive engines) and in gas turbines, the hot reservoir is in reality replaced by the combustion of the air-fuel mixture. Hence, heat sources are realized by indefinitely repeated combustion processes, that is, by irreversible chemical transformations. Finally, engine operation involves friction between mechanical parts in contact, which also implies the irreversible dissipation of work into heat.

6. Irreversibility, Disorder, and Entropy

The irreversibility of real machines can also be verified as follows. The inversion of a reversible engine cycle is an ideal refrigerator. In practice, it is impossible to build a refrigerator by reversing all the mechanical motions and heat flows of an engine, precisely because real engines are irreversible.

Irreversibility implies that real machines are *repetitive* rather than *cyclical*, since the final state of a real cycle differs from the initial one.

Apart from serving as the theoretical foundation of heat engine operation, the theory of heat engines has highlighted the central role of irreversible processes in thermodynamics. While it is possible to convert a given amount of mechanical energy entirely into heat transferred to the environment (or, in general, to a thermal reservoir), it is not possible to convert all the heat extracted from a reservoir entirely into mechanical energy. As we have seen, the irreversibility of the conversion of mechanical energy into heat is physically equivalent to the irreversibility of heat exchange.

By analyzing friction, we see that the conversion between work and heat is observed at the "macroscopic" scale, whereas at the microscopic or molecular scale, ordered and disordered forms of energy are transformed into each other. For instance, the kinetic energy of a moving piston is "ordered," since all the parts of the piston - in the limit, all of its atoms - move together (ignoring local microscopic motions such as atomic vibrations). Friction converts amounts of ordered energy into thermal energy, which is the macroscopic manifestation of molecular energy. The motions of atoms and molecules are chaotic; hence, thermal energy is a disordered form of energy. Disordered states of a physical system are generally more probable than ordered ones; therefore, it is easier to carry out a transformation from an initial state to a less ordered final state than to a more ordered one - that is, at the macroscopic scale, to transform work into heat rather than the reverse. Irreversibility can thus be understood as arising from transitions from "more ordered" and less probable states to "less ordered" (or "more disordered") and more probable states.

Transferring heat to a body (excluding thermal reservoirs) increases both its temperature and its molecular chaos; therefore, a higher temperature means greater molecular disorder, and a lower temperature means the opposite.

Absolute zero [1] would then be the temperature of a completely ordered system; since molecular energy is disordered, perfect order is possible only if all atoms or molecules are motionless. Hence, in classical physics, absolute zero is the state of total immobility of matter (in quantum mechanics, it is merely the minimum-energy state). See also the endnote.

We are therefore led to conclude that "spontaneous" processes generate disorder; that is, in the absence of external work, a thermally isolated system evolves toward more disordered states, and the final equilibrium state of an isolated system cannot be more ordered than the initial one. In short, the disorder of the entire universe increases with time. This rule may seem not to apply to artificial processes such as those analyzed above, but in nature no "artificial" processes exist, since physical laws are universal and do not distinguish between "natural" and "artificial" processes. The production of work by real machines [2] also obeys the law of the irreversible increase of disorder applied to the universe as a whole, which in practice means the union of a physical system (the working fluid, or in this case the entire machine) with its environment. This is the conceptual root of the Second Law of Thermodynamics, of which the Kelvin-Planck and Clausius statements are particular formulations.

However, the foregoing exposition of the Second Law is largely incomplete, chiefly because of the imprecision of the concept of disorder: we have not yet defined how to measure it.

The measure of disorder is defined by a physical quantity known as **entropy**. The entropy of a thermally and mechanically isolated physical system cannot decrease: it increases in irreversible transformations and remains constant in reversible ones. The increase of entropy distinguishes the past from the future, since - for an isolated system - the future entropy will be greater than the present one. The Second Law states that the entropy of the universe must increase over time, and its partial formulations as the Kelvin-Planck and Clausius statements - although apparently limited to specific phenomena and technical problems - already embody its physical meaning.

While the First Law of Thermodynamics (the principle of conservation of total energy) is expressed as an equation - and therefore neither defines the direction in which a physical process evolves nor introduces any temporal asymmetry - the Second Law implies a fundamental temporal asymmetry. According to this principle, the universe as a whole cannot return to any past state (even though cyclic transformations exist, they are strictly local). In other words, the sequence of instants into which we conceptually divide time cannot be reversed. Thus, the Second Law appears to define the direction of time, commonly known as the **"arrow of time."** From a physical standpoint, the profound interest in the Second Law stems from the statistical nature of irreversibility. Because of its statistical character, this concept applies only to many-particle systems. Its connection to order and disorder makes sense only when referring to macroscopic ensembles of identical particles. The Second Law effectively imports the mathematical law of large numbers into physics; consequently, it is fundamentally different from the deterministic laws of classical dynamics and the First Law itself, and it cannot be reduced to a mere mechanical principle [3].

Endnotes

[1] Absolute zero represents the absolute lower limit of any temperature scale rather than a reachable minimum, as no finite sequence of refrigeration cycles can achieve this exact point (in accordance with the Third Law of Thermodynamics), whereas any higher temperature is theoretically attainable.

The temperature scales currently in use are the Kelvin (absolute) scale, the Celsius scale, and the Fahrenheit scale. The first two are universal, but according to the International System of Units (SI), the base unit for thermodynamic temperature is the **kelvin** (K), not the degree Celsius (°C), which is more common in everyday practice. Thermodynamic temperature, expressed in kelvins, is referred to as "absolute temperature" and is the standard in physics because it is directly proportional to the average kinetic energy of atoms and molecules. Naturally, 0 K corresponds to absolute zero. By convention, the triple point of water is defined as exactly 273.16 K or 0.01 °C. Temperature intervals (differences) are identical in the Kelvin and Celsius scales, which differ only in the values assigned to their fixed points. If T denotes temperature in kelvins and t in degrees Celsius, the relationship is $t = T - 273.15$; thus, absolute zero corresponds to -273.15°C.

The Fahrenheit scale is commonly used in the United States; its unit is the degree Fahrenheit (°F). If F denotes the temperature in degrees Fahrenheit, the conversion from the absolute temperature T (in kelvins) is $F = 1.8T - 459.67$. Thus, absolute zero is located at -459.67°F.

[2] "Real" machines operate through irreversible transformations, which generate an increase in entropy. "Ideal" machines are reversible systems, for which the total entropy remains constant. The two statements of the Second Law apply to both, meaning they do not strictly *require* an increase in entropy (since reversible processes are theoretically permitted). However, an entropy increase inevitably occurs because all real machines are strictly irreversible. Conversely, the irreversibility of real engines directly implies $\eta < 1$.

[3] Newton's laws of mechanics imply that all fundamental motions are reversible. Classical dynamics, as well as Special and General Relativity, are time-symmetric. (Such macroscopic theories must assume that the entire universe evolved from initial conditions far from equilibrium to explain the observed arrow of time). The situation is less clear in Quantum Mechanics: the collapse of the wave function ψ is generally considered irreversible, whereas the underlying formal system (the equations and their solutions) is invariant under time reversal (T-symmetry). Thus, almost all commonly accepted fundamental theories of physics are intrinsically time-symmetric at the microscopic level.

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